American Journal of Sciences and Engineering Research

E-ISSN– 2348–703X, Volume 8, Issue 5, 2025

Prediction of Compressive Strength of Calcined Clay-Saw Dust Ash-Cement Concrete Using Scheffe's Simplex Method

Emmanuel Onyemauchechukwu Iwuagwu[1], Lewechi Anyaogu[2] and Julius Okezuo Onyeka[3]

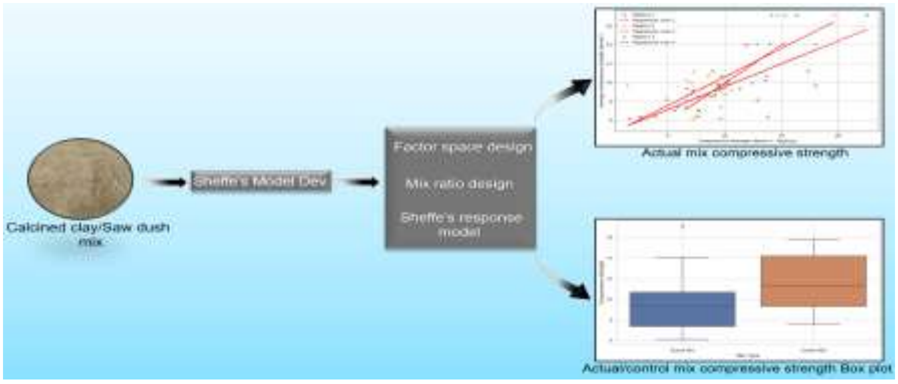
[1]*Department of Civil Engineering, Imo State University Owerri, P.M.B 2000, Owerri, Imo State, Nigeria.*

[2]*Department of Civil Engineering, Federal University of Technology, P.M.B 1526, Owerri, Imo State, Nigeria.*

Abstract: *The high cost of construction materials like cement and reinforcement bars, has led to increased cost of construction. This coupled with the quest for sustainable construction materials has necessitated a search for an alternative binder that can be used solely or in partial replacement of cement in concrete production. Disposing of agricultural waste materials such as sawdust from saw mills, rice husks, groundnut husks, com cob, coconut shells, etc. has constituted an environmental challenge, hence the need to convert them into useful materials to minimize their negative effect on the environment. This study focused on developing a mathematical model for predicting the compressive strength of calcine clay-saw dust ash-cement concrete using Scheffe's simplex method. A total of one hundred and twenty-six (126) cubes were cast, consisting of three cubes per mix ratio and forty-two (42) mix ratio. The first twenty-one (21) mixes were used to develop the model, while the other twenty-one were used to validate the model. The computer program was written for Scheffe's simplex model, using Visual Basic 6.0. The written program predicted the compressive strength for a given mix ratio. The mathematical model results compared favourably with the experimental data and the predictions from the model were tested with the statistical t-test. They were found to be adequate at a 95% confidence level. The optimum compressive strength of the blended concrete at twenty-eight (28) days was found to be 17.11Nmm². The corresponding mixture ratio is as follows: water 0.34, cement 0.503, clay 0.041, saw dust ash 0.056, sand 1.246, granite 2.39. The study proved that calcined clay-saw dust ash can be used effectively as a pozzolanic cementitious material in concrete.*

Keywords: *Calcined clay, Sawdust ash, Compressive strength, Concrete, Scheffe's simplex method.*

Graphical Abstract



 Received-10-10-2025 Accepted- 20-10-2025

I. Introduction

Concrete is one of the crucial materials for infrastructure development due to its versatile application, globally its usage is second to water (Mehta & Monteiro, 2014). For the last few decades, the construction industry has been facing significant environmental challenges due to the high carbon emissions associated with cement production, and the extraction of natural resources for cement production can lead to the depletion of minerals and rocks (Hossain et al., 2019; Gencel et al., 2021). The. The manufacture of one ton of Portland cement (PC) generates about one ton of CO_2, into the atmosphere which constitutes 5% of global CO_2, emissions (IPCC, 2021; Levers et al., 2020). To build a sustainable environment, it is imperative to control CO2 emissions, especially given the rising costs of conventional building materials and environmental hazards (Meyer, 2009; Dhir et al., 2018). More so, the disposal of agricultural waste materials, such as sawdust, rice husk, groundnut husk, corn cob, and coconut shell, presents additional environmental challenges (Bai et al., 2019; Thakur et al., 2021), Converting these waste materials into useful products can mitigate their negative environmental impacts (Nehdi et al., 2018; Sharma et al., 2020). In addressing environmental problems and economic advantages, mixtures of Portland cement (PC) and pozzolans are very commonly used in concrete production (Dhanasekaran et al., 2022; ASTM C618, 2022).

However, adding calcine clay and saw dust ash increases the concrete component from four to six. This makes the orthodox method of mix design, which is used in predicting the properties of concrete such as compressive strength more tedious (Khatib et al., 2021). The problem of identifying the optimum concrete mix becomes very complicated and complex. This aligns with the findings of Khatib et al. (2021), who noted that "mix design represents a multi-criteria optimization challenge, necessitating effective strategies for its resolution." Using the orthodox method of developing mix designs will require carrying out several trials on various mix proportions in the laboratories making it even more difficult to identify optimum concrete mix" With the development of a mathematical model that will predict optimum concrete mix values of Compressive Strength of concrete and other desired properties of concrete, it becomes easier to identify an optimum concrete mix. Developing a mathematical model to predict the optimum concrete mix values for compressive strength and other desired properties can simplify the process and reduce the need for extensive laboratory trials (Zhang et al., 2020; Ranjbar et al., 2019).

Recent applications of Scheffe's method in concrete research demonstrate its effectiveness in optimizing compositions. For instance, Yuan et al. (2018) applied the simplex method to develop predictive models for compressive strength in concrete containing fly ash and silica fume. Their study demonstrated that the method effectively identified optimal material proportions, resulting in significant improvements in compressive strength. Research has shown that using Scheffe's simplex method is particularly advantageous in multi-component systems, where several materials are combined. For instance, Singh and Goyal (2021) explored the effects of different types of pozzolans in concrete and applied Scheffe's method to optimize the blend. The authors found that this approach not only predicted compressive strength accurately but also led to significant cost savings by reducing cement usage (Kumar et al., 2022).

Saw dust has, however, attracted more attention due to environmental pollution and an increasing interest in the conservation of energy and resources (Kumar & Bansal, 2021). This work offers an ideal method to integrate and utilize some waste materials, which are socially acceptable, easily available, and economically within the buying powers of an ordinary man. The presence of such materials in cement concrete not only reduces carbon dioxide emissions but also imparts significant improvement in workability and durability (Bai et al., 2019).

In this study, the effect of using calcine clay and saw dust ash as a partial replacement for cement was investigated. A mathematical model was deployed to predict the compressive strength for all mixed calcined clay-saw dust ash-cement concrete proportions combinations. The model thus predicted an optimum compressive strength for the calcined clay-sawdust ash-cement concrete at 28 days which were statistically validated. This confirms that both calcined clay and sawdust ash exhibit cementitious properties suitable as partial replacements for ordinary Portland cement. The findings of this study show that the proposed model in this study can predict the compressive strength for any combination of mixed proportions of calcined clay-sawdust ash-cement concrete and estimate compressive strength for specific mix proportions. The study demonstrated that calcined clay-sawdust ash can be effectively utilized as a pozzolanic cementitious material in concrete which holds promise to revolutionize the cement industry.

II. Materials and Method

2.1 Materials

In this research work, the materials used were sourced locally within the City of Imo State Metropolis. These materials include Ordinary Portland cement, Sharp River sand, Granite chippings, Calcine Clay Soil, Sawdust Ash, and Water. Dangote^{TM} brand of ordinary Portland cement which conforms to the requirements of BS12:1978 (British Standards Institution, 1978), was used in this work. The aggregates used in this research work were of two sets: The fine aggregate used for this research work was obtained from a flowing river (Otamiri River) but was purchased at the aggregate market at km 1, Aba Road Owerri, Imo State, Nigeria. It was washed and sun-dried for seven days in the laboratory before usage. The maximum diameter of sand used is 5mm. The coarse aggregate used for this Research were granite chippings obtained from the Abakaliki quarry site but purchased at the aggregate market km l. Aba Road Owerri, Imo State. The granite chippings are a maximum size of 20 mm. They were washed and sun-dried for seven days in the laboratory to ensure that they were free from excessive dust, and organic matter. Calcined clay was obtained from Umuasua Autonomous Community in Isiukwuato Local Government Area of Abia State, Nigeria. The material was first ground and then dried in a furnace to remove natural moisture, just like Sawdust ash, the calcined clay was allowed to cool, thereafter, it was sieved with a 150μm sieve aperture to obtain the finest particle of material which approximates the fineness of that of cement used. Sawdust ash was obtained from the timber milling market at Naze Timber Market, Owerri, Imo State, Nigeria. This material was first dried to remove the natural moisture. The waste was burnt in an enclosure (i.e. open drum) at the temperature of about 800-900°C to obtain sawdust ash. The ash was allowed to cool, thereafter the ash was sieved with a 150 μm sieve aperture to obtain the finest particle of material which approximates the fineness of the cement used.

2.2 Methods

The methods employed in this study included experimental methods and mathematical model development.

2.2.1 Simplex Design

This work considered the use of simplex design and regression in the formulation of concrete design models in detail. However, Scheffe's method of optimization was used in the modeling and optimization.

2.2.2 Six Components Factor Space

A factor space is a one-dimensional (a line), a two-dimensional (a plane), a three-dimensional (a tetrahedron), or any other imaginary space where mixture components interact. The boundary with which the mixture components interact is defined by space. Scheffe (1958) stated that (q - 1) space would be used to define the boundary where q components are interacting in a mixture. In other words, a mixture comprising q components can be analyzed using a (q - 1) space.

This research work deals with a six-component concrete mixture. The components that form the concrete mixture are water/cement (w/c) ratio, cement, calcine clay, saw dust ash, river sand, and granite. The number of components, q, is equal to six. The space to be used in the analysis will be q - 1 which is equal to five-dimensional factor space. A five-dimensional factor space is an imaginary dimension space. The imaginary space used is shown in Figure1.

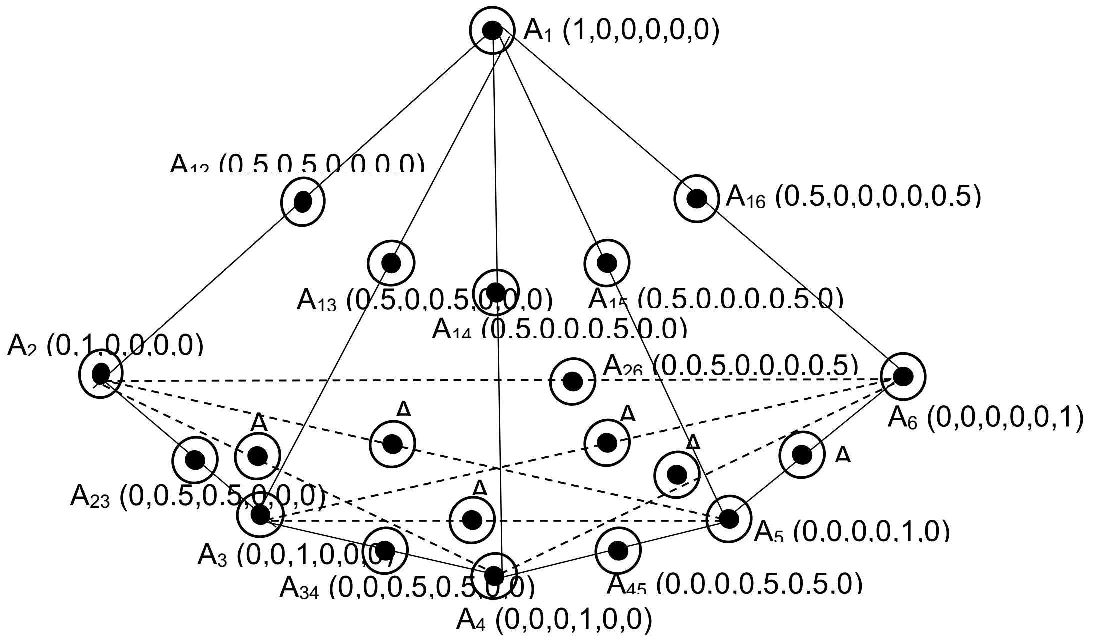


Figure 1: Five-Dimensional Factor Space

The mixture has six components; water, cement, calcine clay, saw dust ash, river sand, and granite, of the i^th component of the mixture such that.

$$X_i \geq 0 \; (i = 1,2,3,4,5,6) \quad (1)$$

And assuming the mixture to be a unit quantity, then the sum of all proportions of the components most be unity. Thus

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 1 \quad (2)$$

This implies that

$$\sum_{i=1}^{q} X_1 = 1 \quad (3)$$

This implies that $0 \leq X_i \leq 1$, the factor space therefore is a regular $(q-1)$ dimensional simplex

In Scheffe's mixture design, the Pseudo components have a relationship with the actual components. This means that the actual component can be derived from the Pseudo components and vice versa. According to Scheffe, Pseudo components were designated as X and the actual components were designated as S. Hence the relationship between X and S as expressed by Scheffe is given in Equation (4).

$$S = A \times X \quad (4)$$

where A is the coefficient of the relationship Equation (4) can thus be transformed into Equation (5) as

$$X = B \times S \quad (5)$$

Equation (5) will be used to determine the actual component of the mixture when the Pseudo components are known, while Equations (3.6) and (37) will be used to determine the Pseudo components of the mixture when the actual components are known. The six components are Water, Cement, Calcine clay, Sawdust ash, Sand and Granite.

Let S_1 = Water, S_2 = Cement, S_3 = Calcine clay, S_4 = Saw dust ash, S_5 = Sand and S_6 = Granite. Then, in keeping with the principle of absolute volume

$$S_1 + S_2 + S_3 + S_4 + S_5 + S_6 = S \quad (6)$$

Or

$$\frac{S_1}{S} + \frac{S_2}{S} + \frac{S_3}{S} + \frac{S_4}{S} + \frac{S_5}{S} + \frac{S_6}{S} = 1 \quad (7)$$

$$where\ \frac{S_i}{S}\ \text{is the proportion of the ith constituent component of the considered concrete mix}.$$

$$\text{Let } \frac{S_i}{S} = Z_i \text{ where i } = 1,2,3,4,5,6 \quad (8)$$

Substituting Equation. (8) into Equation (7), we have

$$Z_1 + Z_2 + Z_3 + Z_4 + Z_5 + Z_6 = 1 \quad (9)$$

According to Henry Scheffe's simplex lattice, the mix ratio drawn in an imaginary space will give 21 points on the five-dimensional factor spaces.

2.2.3 Responses

The responses or dependent variables, Y_i, which are the performance criteria for optimization sought is the compressive strength of calcine clay-sawdust ash cement concrete. The response is presented using a polynomial function of Pseudo components of the mixture.

Simon (2003) derived the Eqn. of response as

$$Y = b_o + \sum b_i X_i + \sum b_{ij} X_i X_{ij} + \sum b_{ijk} X_i X_j X_k + \cdots + e \quad (10)$$

Where b_i, b_{ij} and b_{ijk} are constants; $X_i, X_j, and\ X_k$ are Pseudo components and e is the random error term, which represents the combined effects of all variables not included in the model.

2.2.4 Coefficients of the Polynomial

The number of coefficients of the polynomial depends on the number of components and the degree of polynomial the designer wants. The last degree of polynomial possible is equal to the number of components.

Let the number of components be q and the number of degrees of polynomial m the least number of components, q in any given mixture is equal to two, hence $2 \leq q \leq \infty$. Let the number of coefficients be K; according to Scheffe,

$$K = \frac{(q + m - 1)}{(q - 1)!\, m!} \quad (11)$$

Therefore, the number of coefficients for six pseudo-component mixtures with two degrees of reaction is 21. This also determines the 21 different mix proportions used for the experiment. The Equation of response, Y, for the six-Pseudo component mixture, can be given as

$$Y = b_o + \sum b_i X_i + \sum b_{ij} X_i X_{ij} + \cdots + e \quad (12)$$

Where $0 \leq \text{i} \leq 6$ and i and j represent points on the factor space. Substituting the values of i and j gives:

$$\begin{aligned} Y = b_o &+ b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + b_6 X_6 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{14} X_1 X_4 + b_{15} X_1 X_5 \\ &+ b_{16} X_1 X_6 + b_{23} X_2 X_3 + b_{24} X_2 X_4 + b_{25} X_2 X_5 + b_{26} X_2 X_6 + b_{34} X_3 X_4 + b_{35} X_3 X_5 \\ &+ b_{36} X_3 X_6 + b_{45} X_4 X_5 + b_{46} X_4 X_6 + b_{56} X_5 X_6 + b_{11} X_1^2 + b_{22} X_2^2 + b_{33} X_3^2 + b_{44} X_4^2 \\ &+ b_{55} X_5^2 + b_{66} X_6^2 + e \quad (13) \end{aligned}$$

Multiplying Equation (2) by b_o, yields

$$b_o X_1 + b_o X_2 + b_o X_3 + b_o X_4 + b_o X_5 + b_o X_6 = b_o \quad (14)$$

Multiplying Equation (3) successively by X_1, X_2, X_3, X_4, X_5 and X_6 and rearranging the product gives equation (15)

$$\left.\begin{aligned} X_1^2 &= X_1 - X_1X_2 - X_1X_3 - X_1X_4 - X_1X_5 - X_1X_6 \\ X_2^2 &= X_2 - X_1X_2 - X_2X_3 - X_2X_4 - X_2X_5 - X_2X_6 \\ X_3^2 &= X_3 - X_1X_3 - X_2X_3 - X_3X_4 - X_3X_5 - X_3X_6 \\ X_4^2 &= X_4 - X_1X_4 - X_2X_4 - X_3X_4 - X_4X_5 - X_4X_6 \\ X_5^2 &= X_5 - X_1X_5 - X_2X_5 - X_3X_5 - X_4X_5 - X_5X_6 \\ X_6^2 &= X_6 - X_1X_6 - X_2X_6 - X_3X_6 - X_4X_6 - X_5X_6 \end{aligned}\right\} \quad (15)$$

Substituting Equation (14) and (15) into Equation (13), yields Equation (16)

$$\begin{aligned} \hat{Y} = X_1(b_o + b_1 + b_1) &+ X_2(b_o + b_2 + b_{22}) + X_3(b_o + b_3 + b_{33}) + X_4(b_o + b_4 + b_{44}) \\ &+ X_5(b_o + b_5 + b_{55}) + X_6(b_o + b_6 + b_{66}) + X_1X_2(b_{12} + b_{11} + b_{22}) \\ &+ X_1X_3(b_{13} + b_{11} + b_{33}) + X_1X_4(b_{14} + b_{11} + b_{44}) + X_1X_5(b_{15} + b_{11} + b_{55}) \\ &+ X_1X_6(b_{16} + b_{11} + b_{66}) + X_2X_3(b_{23} + b_{22} + b_{33}) + X_2X_4(b_{24} + b_{22} + b_{44}) \\ &+ X_2X_5(b_{25} + b_{22} + b_{55}) + X_2X_6(b_{26} + b_{22} + b_{66}) + X_3X_4(b_{34} + b_{33} + b_{44}) \\ &+ X_3X_5(b_{35} + b_{33} + b_{55}) + X_3X_6(b_{36} + b_{33} + b_{66}) + X_4X_5(b_{45} + b_{44} + b_{55}) \\ &+ X_4X_6(b_{46} + b_{44} + b_{66}) + X_5X_6(b_{56} + b_{55} + b_{66}) + e \quad (16) \end{aligned}$$

Equation (16) can be expressed in the following form

$$\widehat{Y} = \sum X_i(b_o + b_{ii} + b_{jj}) + \sum X_iX_j(b_{ij} + b_{ii} + b_{jj}) + \cdots + e \quad (17)$$

Let $\propto_i = (b_o + b_{ii} + b_{jj})$ *and* $\propto_{ij} = (b_{ij} + b_{ii} + b_{jj})$, substituting in Equation (18), yields

$$\widehat{Y} = \propto_1 X_1 + \propto_2 X_2 + \propto_3 X_3 + \propto_4 X_4 + \propto_5 X_5 + \propto_6 X_6 + \propto_{12} X_1X_2 + \propto_{13} X_1X_3 + \propto_{14} X_1X_4 + \propto_{15} X_1X_5 + \propto_{16} X_1X_6 + \propto_{23} X_2X_3 + \propto_{24} X_2X_4 + \propto_{25} X_2X_5 + \propto_{26} X_2X_6 + \propto_{34} X_3X_4 + \propto_{35} X_3X_5 + \propto_{36} X_3X_6 + \propto_{45} X_4X_5 + \propto_{46} X_4X_6 + \propto_{56} X_5X_6 + e \quad (18)$$

Where e = standard error, $\propto_1$ and $\propto_{1j}$ are the coefficient of response equation and pseudo components of the mix respectively.

Equation. (18) can be expressed in the form

$$\widehat{Y} = \sum_{i=1}^{6} \propto_i X_i + \sum_{i \le j \le 6}^{6} \propto_i X_iX_{ij} \quad (19)$$

Equation (19) is the response of the pure component "i" and the binary component "ij"

If the response function is represented by y, the response function for the pure component and that for the binary mixture components will be y_i and y_{ij} respectively.

$$y_i = \sum_{i=1}^{q} \propto_i X_i \quad (20)$$

$$y_{ij} = \sum_{i=1}^{6} \propto_i X_i + \sum_{i \le j \le 6}^{6} \propto_i X_iX_{ij} \quad (21)$$

If the response at i^th point on the factor space is y_i, then at point 1, component X_1 = 1 and components X_2, X_3, X_4, X_5 and X_6, are all equal to zero at X_1 = 1 Equation (22) becomes

$$y_1 = \propto_1 \quad (22)$$

Substituting X_2 = 1 and $X_1 =, X_3 = X_4 = X_5 = X_6 = 0$, Equation (20) yields Equation (23) becomes

Similarly, Eqn. (24) and (25) can be expressed in the form

$$y_i = \propto_i \quad (23)$$

For point, ij of the factor space, component $X_i = \frac{1}{2}$; $X_j = \frac{1}{2}$ and $X_3 = X_4 = X_5 = X_6 = 0$, Equation (21) gives

$$y_{ij} = \frac{1}{2}\propto_i + \frac{1}{2}\propto_j + \frac{1}{4}\propto_{ij} \quad (24)$$

Rearranging Equation (23) and (24) yields Equation (25) and (26)

$$\propto_i = y_1 \quad (25)$$

$$\propto_{ij} = 4y_{ij} - 2\propto_i - 2\propto_i \quad (26)$$

Let $\propto_i = y_i$ and $\propto_j = y_j$, hence, Equation (25) and (26) will yield

$$\propto_i = y_1 \quad (27)$$

$$\propto_{ij} = 4y_{ij} - 2\propto_i - 2\propto_i \quad (28)$$

Substituting Equation (27) and (28) into Eqn. (18) and simplifying the result, yields Equation (28)

$$Y = X_1y_1(2X_1 - 1) + X_2y_2(2X_2 - 1) + X_3y_3(2X_3 - 1) + X_4y_4(2X_4 - 1) + X_5y_5(2X_5 - 1) + X_6y_6(2X_6 - 1) + 4y_{12}X_1X_2 + 4y_{13}X_1X_3 + 4y_{14}X_1X_4 + 4y_{15}X_1X_5 + 4y_{16}X_1X_6 + 4y_{23}X_2X_3 + 4y_{24}X_2X_4 + 4y_{25}X_2X_5 + 4y_{26}X_2X_6 + 4y_{34}X_3X_4 + 4y_{35}X_3X_5 + 4y_{36}X_3X_6 + 4y_{45}X_4X_5 + 4y_{46}X_4X_6 + 4y_{56}X_5X_6 \quad (29)$$

Equation (29) is the mixture design mode for the optimization of a concrete mixture consisting of six components. The term, y_i and y_{ij} are the responses (representing compressive strength) at the points i and ij. These responses are determined by carrying out compressive tests on cubes using Calcine Clay and Sawdust ash as the components of concrete.

2.2.5 Concrete Mix Ratios

The starting set of actual components S and pseudo components X used in this research for Six mixed ratios that defined the vertices of the five-dimensional simplex lattice are shown in Table 1:

Table 1: Six Mix Ratios (Actual and Pseudo) Obtained from Scheffe's (6, 2) Factor Space

Actual Mix ratios							Pseudo Mix ratios					
Water S_1	Cement S_2	CC S_3	SDA S_4	Sand S_5	Granite S_6	Response	X_1	X_2	X_3	X_4	X_5	X_6
0.50	0.95	0.02	0.03	2.00	4.00	Y_1	1	0	0	0	0	0
0.52	0.90	0.04	0.06	1.98	3.80	Y_2	0	1	0	0	0	0
0.55	0.85	0.06	0.09	2.15	3.50	Y_3	0	0	1	0	0	0
0.56	0.80	0.08	0.12	1.88	4.20	Y_4	0	0	0	1	0	0
0.60	0.78	0.09	0.13	2.20	4.15	Y_5	0	0	0	0	1	0
0.65	0.75	0.12	0.13	2.25	4.25	$Y_{\wedge}$	0	0	0	0	0	1

As stated in Equation (4), the actual mix ratios relate to pseudo mix ratios in defined by the following equation

$$S = A \times X$$

where S, A, and X represent the Actual mix ratio, coefficient of relation matrix, and pseudo mix ratio respectively. According to Osadebe et.al (2008), matrix A can be taken to be the transpose of the first six real mix ratios shown in Table 1, and this resulted in matrix A.

$$[A] = \begin{bmatrix} 0.5 & 0.52 & 0.55 & 0.58 & 0.60 & 0.67 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.78 & 0.75 \\ 0.02 & 0.04 & 0.06 & 0.08 & 0.09 & 0.12 \\ 0.03 & 0.06 & 0.09 & 0.12 & 0.13 & 0.13 \\ 2.00 & 1.98 & 2.15 & 0.88 & 2.20 & 2.25 \\ 4.00 & 3.80 & 3.50 & 4.20 & 4.15 & 4.25 \end{bmatrix} \quad (30)$$

The Six actual and pseudo mix ratios in Table 1 correspond to points of observations, A_1, A_2, A_3, A_4, A_5, A_6 located at the Six vertices of the five-dimensional simplex lattice. For a (6, 2) simplex design, fifteen other observations are needed to add up to the first six to get a total of twenty-one observations needed for the development of the response function. The remaining fifteen points are located at the midpoints of the lines joining the Six vertices. On successive substitution of these fifteen pseudos mix ratios into Equation (4), the Actual mix ratios corresponding to the pseudo-ones were obtained. Their values are shown in Table 2.

Let S_1 = Water, S_2 = Cement, S_3 = Calcine clay, S_4 = Saw dust ash, S_5 = Sand and S_6 = Granite for the Actual mix ratio and X_1 = Water, X_2 = Cement, X_3 = Calcine clay, X_4 = Saw dust ash; X_5 = Sand and X_6 = Granite for the pseudo mix ratio.

Table 2: Actual and Pseudo Components of the Actual mix Ratio

Actual Mix ratios							Pseudo Mix ratios					
S_1	S_2	S_3	S_4	S_5	S_6	Response	X_1	X_2	X_3	X_4	X_5	X_6
0.500	0.950	0.020	0.030	2.000	4.000	A1	1	0	0	0	0	0
0.520	0.900	0.040	0.060	1.980	3.800	A2	0	1	0	0	0	0
0.550	0.850	0.060	0.090	2.150	3.500	A3	0	0	1	0	0	0
0.580	0.800	0.080	0.120	1.880	4.200	A4	0	0	0	1	0	0
0.600	0.780	0.090	0.130	2.200	4.150	A5	0	0	0	0	1	0
0.650	0.750	0.120	0.130	2.250	4.250	A6	0	0	0	0	0	1
0.510	0.925	0.030	0.130	2.000	4.000	A12	0.5	0.5	0	0	0	0
0.525	0.900	0.040	0.045	1.980	3.800	A13	0.5	0	0.5	0	0	0
0.540	0.875	0.050	0.060	2.150	3.500	A14	0.5	0	0	0.5	0	0
0.550	0.865	0.055	0.075	1.880	4.200	A15	0.5	0	0	0	0.5	0
0.575	0.850	0.070	0.080	2.200	4.150	A16	0.5	0	0	0	0	0.5

0.535	0.875	0.050	0.080	2.250	4.250	A23	0	0.5	0.5	0	0	0
0.550	0.850	0.060	0.075	2.000	4.000	A24	0	0.5	0	0.5	0	0
0.560	0.840	0.065	0.090	1.980	3.800	A25	0	0.5	0	0	0.5	0
0.585	0.825	0.080	0.095	2.150	3.500	A26	0	0.5	0	0	0	0.5
0.565	0.825	0.070	0.105	1.880	4.200	A34	0	0	0.5	0.5	0	0
0.575	0.815	0.075	0.110	2.200	4.150	A35	0	0	0.5	0	0.5	0
0.600	0.800	0.090	0.110	2.250	4.250	A36	0	0	0.5	0	0	0.5
0.590	0.790	0.085	0.125	2.000	4.000	A45	0	0	0	0.5	0.5	0
0.615	0.775	0.100	0.125	1.980	3.800	A46	0	0	0	0.5	0	0.5
0.625	0.765	0.105	0.130	2.150	3.500	A56	0	0	0	0	0.5	0.5

2.2.6 Control Points

Another set of twenty-one proportions are required to confirm the adequacy of the model of Equation (29). The set of mixture proportions are called control mixture proportions. Therefore, twenty-one control points will be used. They are $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{17}, C_{18}, C_{19}, C_{20}, C_{21},$

Table 3: Actual and Pseudo Components of the Twenty-one Control Points of Observation

Actual Mix ratios							Pseudo Mix ratios					
S1	S2	S3	S4	S5	S6	Response	X1	X2	X3	X4	X5	X6
0.550	0.700	0.200	0.100	2.440	4.600	C1	0.330	0.330	0.330	0.000	0.000	0.000
0.580	0.750	0.120	0.130	2.200	4.300	C2	0.330	0.000	0.330	0.330	0.000	0.000
0.600	0.800	0.090	0.110	2.000	4.100	C3	0.330	0.000	0.000	0.330	0.330	0.330
0.620	0.850	0.050	0.100	1.890	3.880	C4	0.250	0.250	0.250	0.250	0.000	0.000
0.640	0.900	0.030	0.070	1.850	3.840	C5	0.250	0.200	0.250	0.250	0.250	0.000
0.660	0.950	0.020	0.030	1.800	3.810	C6	0.250	0.000	0.000	0.250	0.250	0.250
0.546	0.856	0.058	0.086	2.042	3.930	C7	0.200	0.200	0.200	0.200	0.200	0.000
0.446	0.666	0.054	0.080	1.642	3.130	C8	0.000	0.200	0.200	0.200	0.200	0.000
0.472	0.636	0.070	0.094	1.696	3.220	C9	0.000	0.000	0.200	0.200	0.200	0.200
0.526	0.890	0.044	0.066	2.002	3.900	C10	0.400	0.200	0.200	0.200	0.000	0.000
0.533	0.886	0.046	0.068	2.047	3.900	C11	0.300	0.400	0.100	0.000	0.200	0.000
0.562	0.840	0.068	0.092	2.086	3.890	C12	0.200	0.000	0.400	0.200	0.000	0.200
0.600	0.799	0.089	0.112	2.190	4.045	C13	0.100	0.000	0.200	0.000	0.300	0.400
0.543	0.860	0.058	0.082	1.973	4.065	C14	0.300	0.200	0.000	0.400	0.000	0.100

0.566	0.830	0.069	0.101	2.118	3.995	C15	0.200	0.000	0.200	0.100	0.500	0.000
0.539	0.870	0.054	0.076	1.983	4.025	C16	0.300	0.300	0.000	0.300	0.000	0.100
0.573	0.838	0.071	0.091	2.148	3.915	C17	0.250	0.000	0.350	0.000	0.100	0.300
0.554	0.850	0.064	0.086	2.008	4.050	C18	0.200	0.300	0.000	0.300	0.000	0.200
0.545	0.873	0.053	0.074	2.105	3.840	C19	0.400	0.000	0.400	0.000	0.100	0.100
0.536	0.888	0.048	0.065	2.016	4.028	C20	0.500	0.200	0.000	0.150	0.000	0.150
0.568	0.843	0.068	0.090	2.138	3.963	C21	0.300	0.000	0.250	0.000	0.250	0.200

2.2.7 Compressive Strength Test

Compressive strength tests were carried out to determine the responses needed to formulate and validate the optimization function. The calcined clay-sawdust-cement concrete specimens were thoroughly mixed with river sand and granite and, water was added. The mixing continued until a uniform and consistent concrete mix was obtained. The entire concrete was cast in concrete cube mould measuring 150 x 150 x 150 x 150 mm in size. A total of 126 cubes were produced from the 42 mix ratios given in Tables 2 and 3, with three cubes from each mix. The first set of 63 cubes made from the first set of 21 mix ratios, was used as a control test for validating the optimization models. The concrete cubes were cured in water for 28 days, and then crushed in a universal testing machine. The compression load at failure was recorded and used in Equation (30) to determine the compressive strength of the calcine clay-sawdust ash-cement concrete and presented in Table 4.

$$f_c = \frac{P}{A} \qquad (30)$$

Where:

f_c = Compressive Strength Test (MPa)

P = Compressive load of the cube at failure (kN)

A = Cross-sectional area of mold (mm^2)

2.2.8 Formulation of Scheffe's Response Function for Optimization of Compressive Strength of Calcine Clay-Sawdust Ash-cement Concrete

The Scheffe's response function for optimization of compressive strength of calcine clay-sawdust ash-cement concrete was formulated by substituting the values of the compressive strength results y_j, from Table 4, into Scheffe's model given Equation (29).

Substituting these values gives Equation (31)

$$Y = 13.93X_1(2X_1 - 1) + 8.11X_2(2X_2 - 1) + 14.04X_3(2X_3 - 1) + 17.10X_4(2X_4 - 1) + 17.11K_5(2X_5 - 1) + 8.75X_6(2X_6 - 1) + 25.32X_1X_2 + 25.60X_1X_3 + 38.84X_1X_4 + 38.64X_1X_5 + 24.44X_1X_6 + 42.72X_2X_3 + 36.52X_2X_4 + 40.72X_2X_5 + 34.00X_2X_6 + 39.64X_3X_4 + 29.48X_3X_5 + 41.24X_3X_6 + 44.72X_4X_5 + 29.68X_4X_6 + 25.28X_5X_6 \qquad (31)$$

Equation (31) is Scheffe's response function for the optimization of compressive strength of calcium clay-sawdust ash cement concrete. The compressive strengths from the Scheffe's response function were calculated using Equation (31). The experimental result values and those obtained from Scheffe's response function are as shown in Table 4.

2.2.9 Test of Goodness of Fit of Scheffe's Response Model

The test for adequacy for Scheffe's Response Model was done using statistics student's t-test at 95% accuracy level. The following two hypotheses were tested. Null Hypothesis: This states that there is no significant difference between the experimental and theoretical (model) results. Alternative Hypothesis: States that there is a significant difference between the experimental and theoretical (model) results. The null hypothesis test was carried out using both student t-tests at a 95% confidence level. The results are as shown in Table 6 using the following equations:

Where:

Y_E = Responses compressive strength from the experiment

Y_M = Responses (compressive strength) from the Second-degree polynomial equation

N = Number of observations points

Di = Difference of Y_E and Y_M

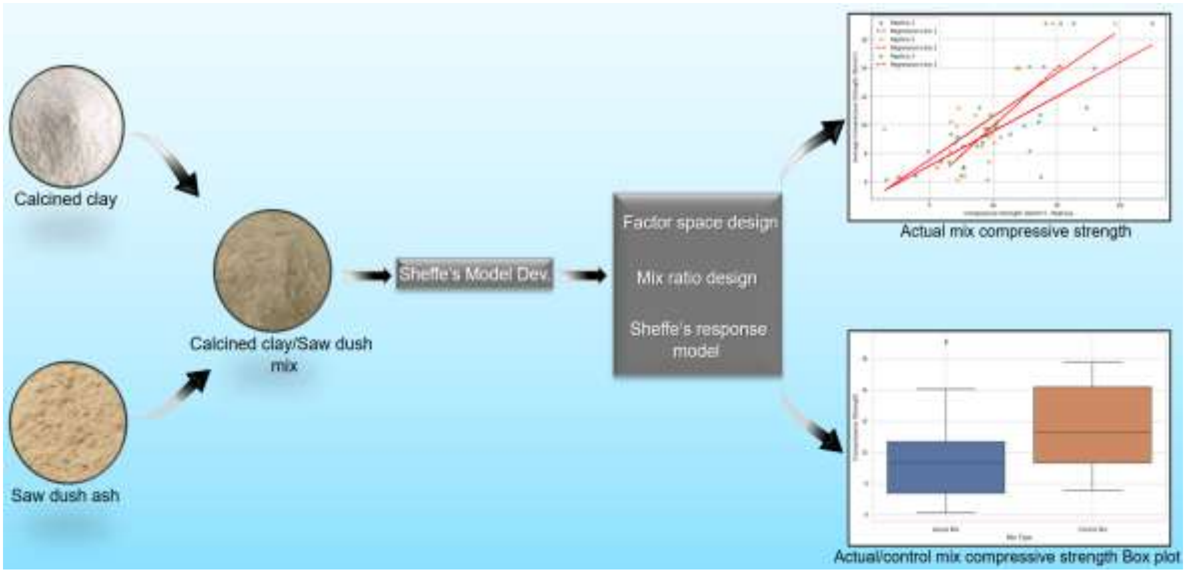
$$\text{Mean of difference of } Y_E \text{ and } Y_M = D_A = \frac{\sum D_1}{N} \quad (32)$$

$$\text{Variance of difference of Di and DA } S^2 = \frac{\sum(D_A - D_I)}{N-1} \quad (33)$$

$$\text{Calculated value of t} = \frac{\mathrm{D_A} - \mathrm{N}^{0.5}}{\mathrm{S}} \quad (34)$$

III. RESULTS AND ANALYSIS

The process of mixing calcined clay and saw dust ash in this study and the subsequent deployment of Scheffe's model is illustrated in Scheme 1.



Scheme 1. Illustration of mixing calcined clay and saw dust ash in this study and the subsequent deployment of Scheffe's model in this study.

The compressive strength of the cubes obtained from the experiment and model are given in Table 4 & 5.

Table 4: Compressive strength (N/mm2) of the 28-day-old concrete cubes for the Actual Mix

Actual Points	Compressive Strength Test (N/mm^2)			Average laboratory Compressive Strength (N/mm^2)	Scheffe's Model Compressive Strength Results (N/mm^2)
	Replica 1	Replica 2	Replica 3		
A1	18.00	12.09	11.82	13.97	13.93
A2	04.89	06.62	12.89	08.13	08.13
A3	14.00	15.29	12.84	14.04	14.04
A4	16.36	19.60	15.33	17.10	17.10
A5	22.53	14.67	14.13	17.11	17.11

A6	09.16	10.08	07.02	08.75	08.76
A12	02.53	02.67	13.78	06.33	06.33
A13	03.91	07.78	07.51	06.40	06.40
A14	10.22	09.33	09.60	09.72	09.72
A15	18.00	01.47	09.51	09.66	09.66
A16	01.60	07.20	09.56	06.12	06.12
A23	13.73	08.58	09.73	10.68	10.68
A24	07.20	10.62	09.60	09.14	09.14
A25	13.60	06.67	10.31	10.19	10.19
A26	07.64	09.11	08.76	08.50	08.50
A34	12.44	07.11	10.22	09.92	09.93
A35	05.91	09.69	06.53	07.38	07.38
A36	11.37	09.91	06.69	09.32	10.32
A45	17.33	07.29	08.93	11.18	11.12
A46	07.69	05.60	07.69	06.99	07.41
A56	07.69	05.60	07.69	06.99	06.32

The linear regression plots for both the actual mix and control mix, showing the relationship between each replica's compressive strength and the average compressive strength were presented in Fig. 2 and 3.

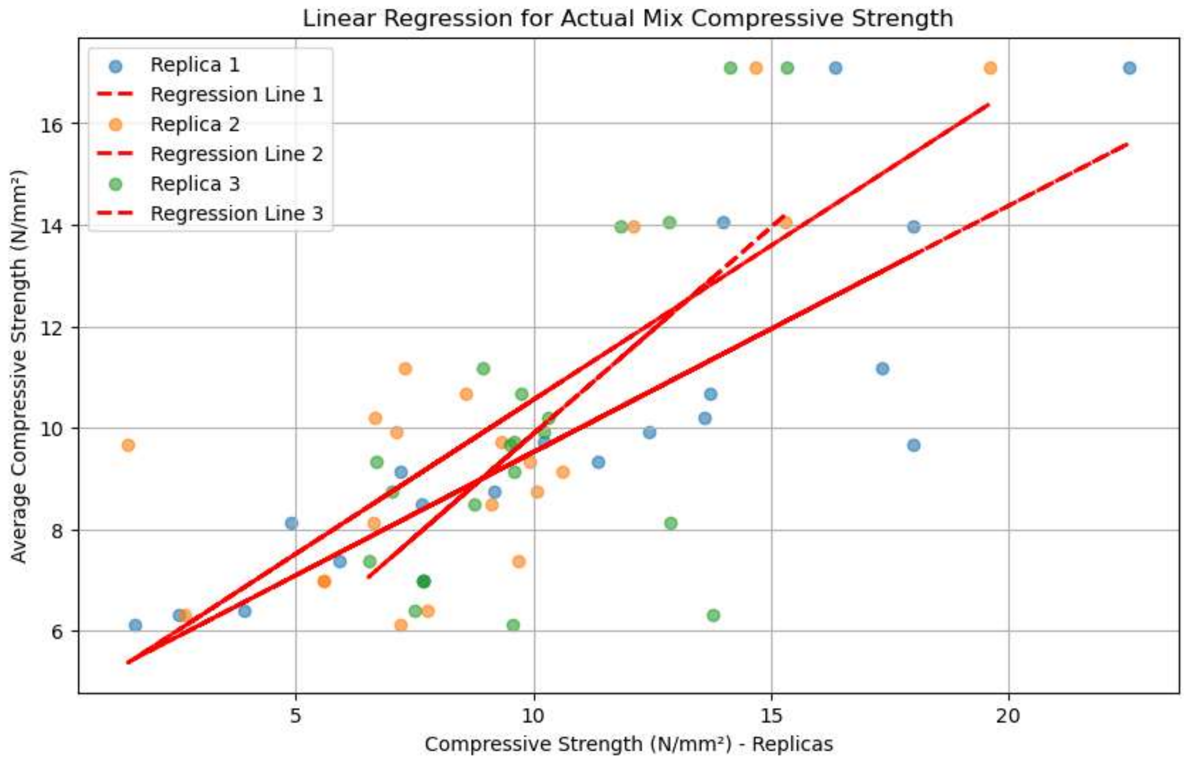


Figure 2: The Graph showing scatter plots and linear regression lines for each replica against the average compressive strength for the Actual Mix.

Table 5: Compressive strength (N/mm^2) of the 28-day-old concrete cubes for the control mix

Control Points	Compressive Strength Test (N/mm^2)			Average laboratory Compressive Strength (N/mm^2)	Scheffe's Model Compressive Strength Results (N/mm^2)
	Replica 1	Replica 2	Replica 3		
C1	14.09	13.91	15.33	14.44	14.44

C2	14.40	11.38	13.16	12.98	11.27
C3	12.00	11.24	13.38	12.21	08.88
C4	14.44	15.69	15.91	15.35	15.39
C5	15.56	09.47	15.11	13.38	13.04
C6	18.27	12.40	14.18	14.95	15.44
C7	10.22	13.87	14.18	12.76	09.33
C8	10.84	13.87	15.47	13.39	08.96
C9	06.49	12.00	07.64	08.71	12.35
C10	11.91	12.84	12.18	12.31	12.29
C11	11.16	12.44	13.11	12.24	09.32
C12	07.20	10.22	10.09	09.17	10.50
C13	11.96	09.29	09.87	10.37	11.91
C14	10.13	11.96	13.56	11.88	10.08
C15	11.38	10.18	06.98	09.51	08.57
C16	08.18	11.64	08.18	09.33	11.02
C17	11.16	11.56	09.69	10.80	15.21
C18	20.31	12.53	13.51	15.45	14.25
C19	07.33	06.62	08.71	07.55	10.52
C20	13.29	19.33	14.71	15.78	16.03
C21	09.20	11.11	12.22	10.84	09.72

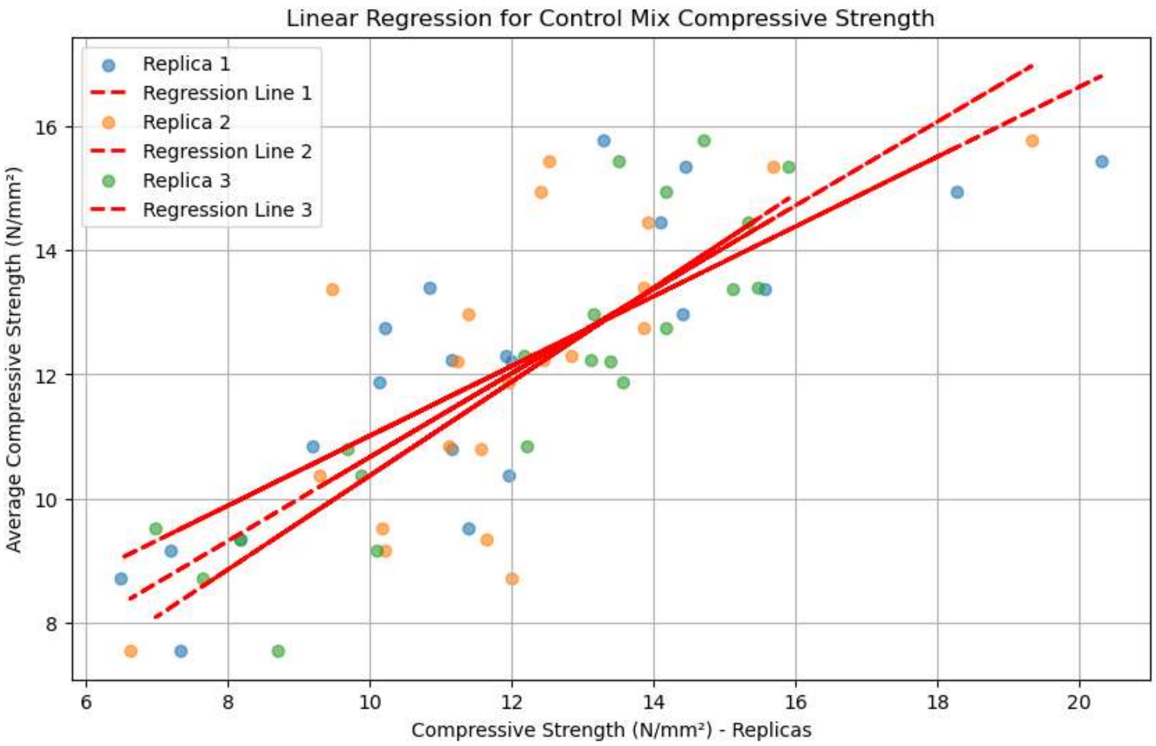


Figure 3: The graph shows scatter plots and linear regression lines for each replica 1-3 against the average compressive strength for the control mix.

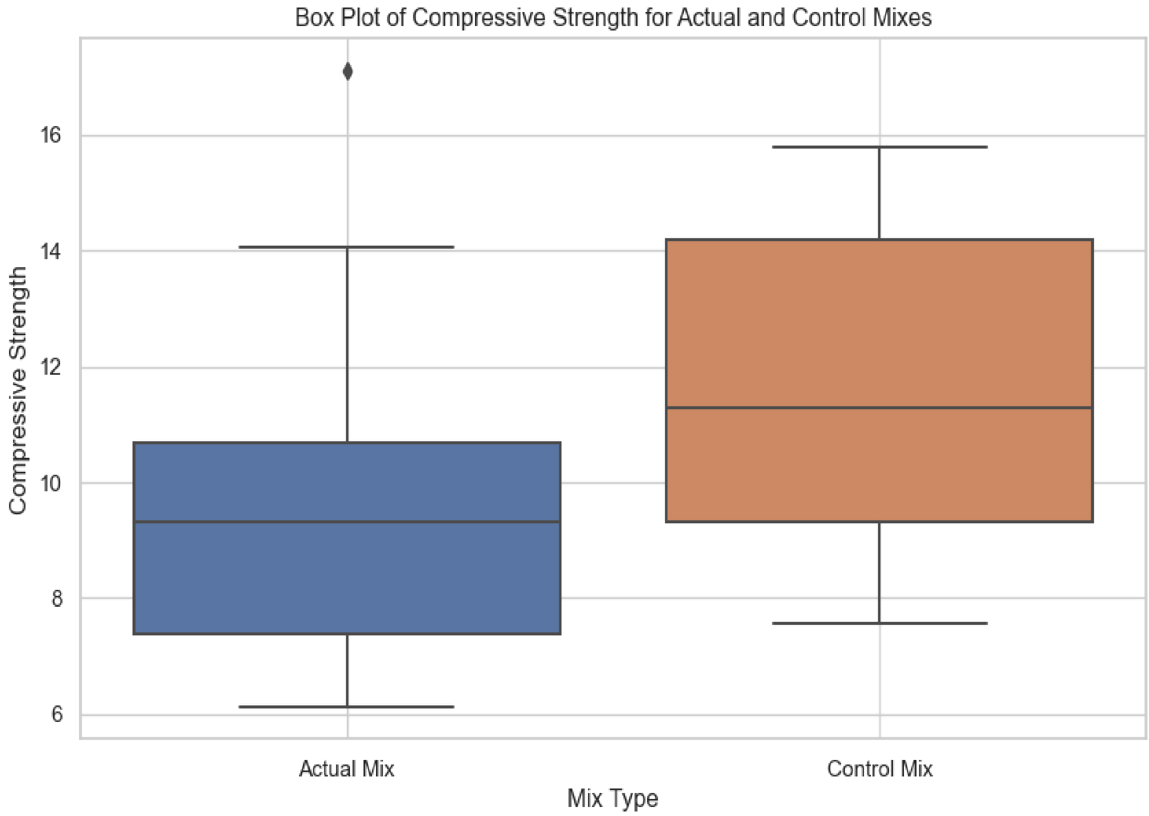


Figure 4: Box Plot of Compressive Strength for Actual and Control Mixes

The compressive strengths of the actual mix and the control mix were compared using the average compressive strength from Table 4 for actual mixes and Table 5 for control mixes. The X-Axis: Mix Type (Actual Mix, Control Mix), Y-Axis: Compressive Strength (N/mm²)

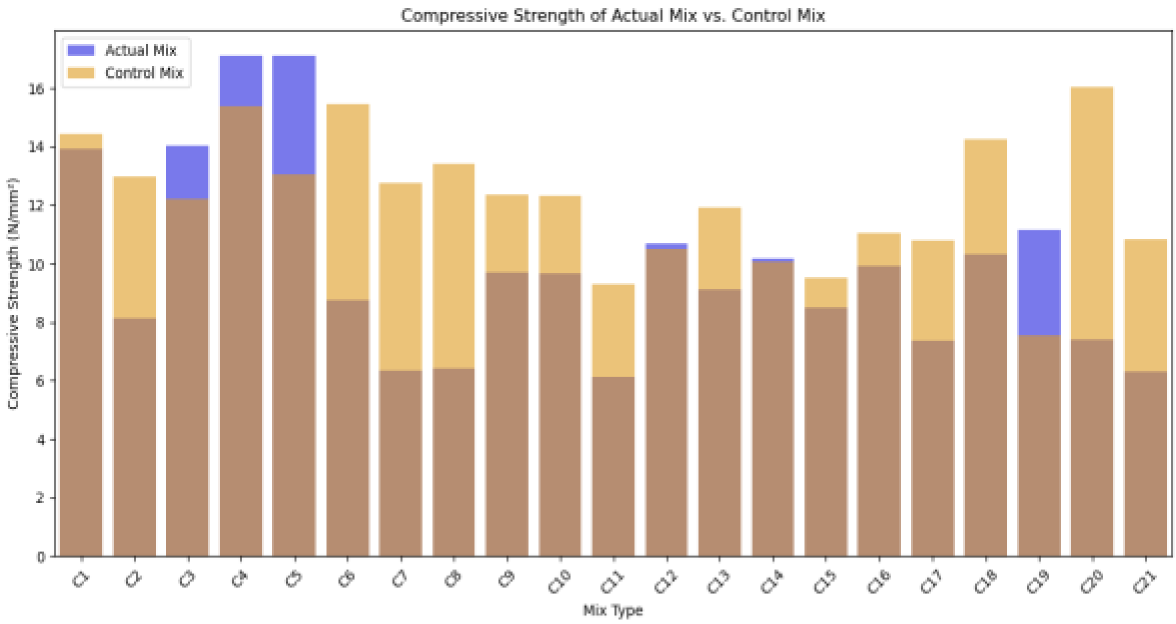


Figure 5: The graph showing the comparison between the average compressive strengths of the actual mix and the control mix

Table 6: Statistical t-test computations for Scheffe's Response Model

Control Points	Y_E	Y_M	$D_A = Y_E - Y_M$	$D_A - D_I$	$(D_A - D_I)^2$
C1	14.44	14.443	-0.003	-0.209	0.0437607
C2	11.27	11.274	-0.004	-0.208	0.0433433
C3	8.69	8.881	-0.191	-0.021	0.0004490
C4	15.33	15.390	-0.060	-0.152	0.0231619
C5	12.33	13.037	-0.707	0.495	0.2448365
C6	14.93	15.444	-0.514	0.302	0.0910890
C7	8.96	9.333	-0.373	0.161	0.0258597
C8	8.71	8.957	-0.247	0.035	0.0012117
C9	12.31	12.350	-0.040	-0.172	0.0296496
C10	12.23	12.294	-0.064	-0.148	0.0219604
C11	9.17	9.323	-0.153	-0.059	0.0035035
C12	10.37	10.501	-0.131	-0.081	0.0065919
C13	11.88	11.907	-0.027	-0.185	0.0342955
C14	9.51	10.079	-0.569	0.357	0.1273130
C15	9.33	8.571	0.759	-0.971	0.9432109
C16	10.8	11.020	-0.220	0.008	0.0000610
C17	15.45	15.211	0.239	-0.451	0.2035728
C18	14.19	14.249	-0.059	-0.153	0.0234673
C19	7.56	10.520	-2.960	2.748	7.5504572
C20	15.78	16.033	-0.253	0.041	0.0016654
C21	10.84	9.719	1.121	-1.333	1.7773968
	$\sum D_I$	11.84		$\sum(D_A - D_I)^2$	11.1968572
		D_A	-0.212		
				S^2	0.559842862
				S	0.748226478
				t	-1.26825859

Allowable total variation in the t-test is obtained as follows:

Degree of freedom = 20
5 % significance for Two-Tailed Test = 0.975
Therefore, allowable total variation in the t-test = $(t_{0.975\times(N-1)})$, $t_{0.975,\times 20}$ = 2.086 (Obtained from standard statistical table). From Table 6, the calculated t = -1.27, Thus, t(table) > t(calculated)

To visualize the significance of the differences/deviation between the predicted and actual compressive strengths, the predicted and actual compressive strengths, are plotted from the Statistical t-test results from Table 6 and presented in Fig. 5.

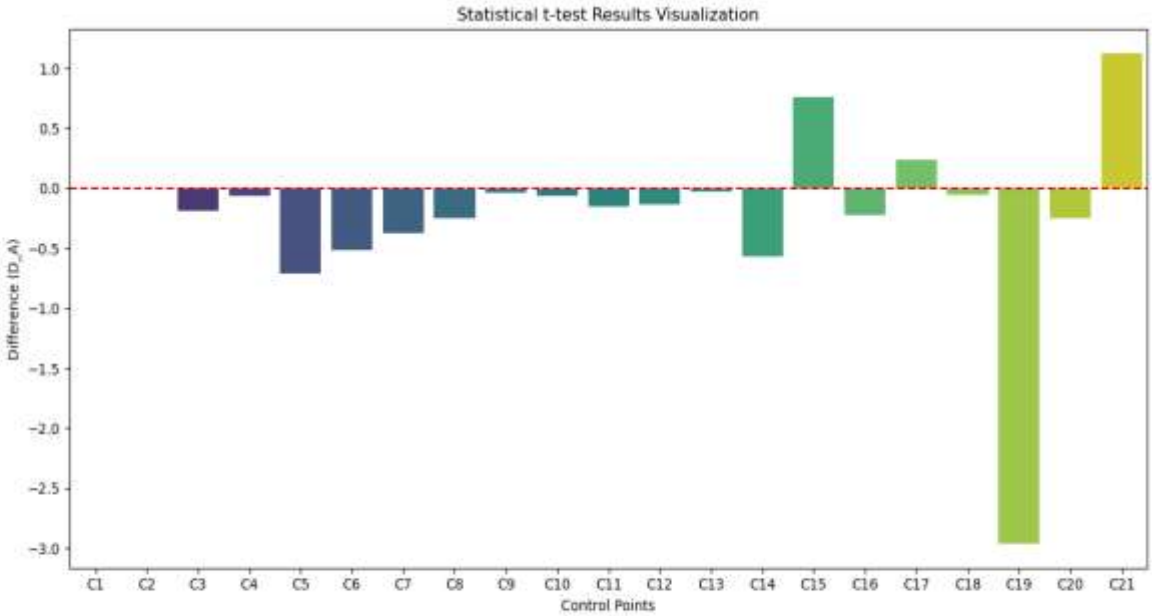


Figure 6: Graph showing the deviation between the predicted and actual compressive strengths.

The mathematical model predicted the compressive strength for all mixed calcined clay-saw dust ash-cement concrete proportions combinations. In the actual mix, the compressive strengths vary significantly among different replicas, with an average strength across points ranging from 6.33 N/mm² (A12) to 17.11 N/mm² (A5), indicating considerable variability in performance across different points. In contrast, the control mix demonstrated a more consistent performance, with average strengths ranging from 7.55 N/mm² (C19) to 15.39 N/mm² (C4). From the Box Plot of Compressive Strength for Actual and Control Mixes in Fig. 3, the first quarter of the actual mix has a spread between 6.12 N/mm² to 7.19 N/mm² with an average range of 1.07 N/mm², the second quarter has a spread between 7.19 N/mm² to 9.32 N/mm² with an average range of 2.13 N/mm², the third quarter has a spread between 9.32 N/mm² to 10.93 N/mm² with an average range of 1.61 N/mm², and the fourth quarter has a spread between 10.93 N/mm² to 17.1 N/mm² with an average range of 6.17 N/mm². This indicates that the second quarter has the smallest spread, and the fourth quarter has the largest spread which suggests that there exists variability in the formulation or execution of the Actual Mix which impacts the overall performance.

For the Control mix, the first quarter has a spread between 7.56 N/mm² to 9.17 N/mm² with an average range of 1.61 N/mm², the second quarter has a spread between 9.17 N/mm² to 11.21 N/mm² with an average range of 2.04 N/mm², the third quarter has a spread between 11.21 N/mm² to 14.19 N/mm² with an average range of 2.98 N/mm², and the fourth quarter has a spread between 14.19 N/mm² to 15.78 N/mm² with an average range of 1.59 N/mm². The Control Mix shows lower variability, suggesting a more reliable formulation and execution.

The model predicts an optimum compressive strength of 17.11 N/mm² for the calcined clay-sawdust ash-cement concrete at 28 days. The corresponding mix ratio is as follows: Water 0.60, Cement 0.78, Calcined Clay 0.09, Sawdust Ash 0.013, Sand 2.20, and Granite 4.15. These predictions were validated at a 95% accuracy level using a student's T-test. As detailed in Table 6, the computed t-value is -1.27, which is less than the t-table value of 2.086 with 20 degrees of freedom (N-1), indicating that the differences between the experimental values (Y_E) and the model predictions (Y_M) are not statistically significant at the 5% significance level. The sums of squares from the t-test indicate that while discrepancies exist between experimental and model values, these variations are relatively minor, affirming the model's validity.

IV. Conclusion

Here, the use of calcined clay and sawdust ash as partial replacements for cement was explored. A mathematical model was employed to predict the compressive strength for all combinations of calcined clay-sawdust ash-cement concrete mixtures. The model successfully predicted an optimal compressive strength for the calcined clay-sawdust ash-cement concrete at 28 days. Statistical analysis was conducted to validate the model predictions. The results obtained in the study confirm that both calcined clay and sawdust ash possess cementitious properties, making them suitable as partial replacements for ordinary Portland cement. It was also established that the model can accurately predict the compressive strength for various mix proportions of calcined clay-sawdust ash-cement concrete and estimate strength for specific mixes. This study also highlights the potential of calcined clay-sawdust ash as a pozzolanic cementitious material, offering significant promise for transforming the cement industry.

V. References

1. ASTM C618. (2022). Standard Specification for Coal Fly Ash and Raw or Calcined Natural Pozzolan for Use in Concrete
2. Bai, Y., Zhang, L., & Zhang, C. (2019). Sustainable use of agricultural waste in concrete production: A review. *Construction and Building Materials*, 214, 658-669.
3. British Standards Institution. (1978). *Specification for Portland cement (BS 12:1978)*. London: BSI.
4. Dhanasekaran, R., Swaminathan, R., & Muthusamy, V. (2022). The role of supplementary cementitious materials in sustainable concrete: A review. *Journal of Cleaner Production*, 331, 129998.
5. Dhir, R. K., Jones, M. R., & McCarthy, M. J. (2018). Sustainable concrete: An overview. *Cement and Concrete Composites*, 92, 202-211.
6. Gencel, O., & Yilmaz, M. (2021). Environmental impacts of concrete: A comprehensive review. *Materials Today: Proceedings*, 44, 220-226.
7. Hossain, K. M. A., Hossain, M. M., & Islam, M. S. (2019). Sustainable concrete: A review of agricultural waste utilization in concrete production. *Journal of Cleaner Production*, 230, 1415-1434.
8. IPCC. (2021). "Climate Change 2021: The Physical Science Basis." Contribution of Working Group I to the Sixth Assessment Report.
9. Meyer, C. (2009). "The greening of the concrete industry." *Cement and Concrete Composites*, 31(8), 601-605.
10. Osadebe, N.N., and Ibearuegbulem, O.M.,(2008). “Application of Osadebe’s Alternative Regression Model in Optimizing Compressive Strength of Periwinkle Shell-Granite Concrete”, NSE Technical Transaction, Vol.43 (1) 47-59
11. Singh, K., & Goyal, M. (2021). Performance evaluation of concrete with different pozzolans using Scheffe’s simplex method. *Construction and Building Materials*, 275, 121073.
12. Yuan, Y., Zhang, Y., & Guo, X. (2018). Predictive modeling of compressive strength for concrete with SCMs using Scheffe’s simplex method. *Construction and Building Materials*, 184, 200-209.
13. Khatib, J. M., Ameer, A. A., & Al-Masry, W. (2021). Multi-objective optimization in concrete mix design: Recent developments and future directions. *Materials Today: Proceedings*, 45, 2354-2360.
14. Khatib, J. M., Soutsos, M. N., & Hossain, K. M. A. (2021). Optimization of concrete mix design using a multi-criteria approach. *Construction and Building Materials*, 283, 122706.
15. Kumar, A., & Bansal, R. K. (2021). Sustainability in construction materials: A review. *Journal of Environmental Management*, 290, 112586.
16. Kumar, V., Singh, K., & Gupta, A. (2022). Economical concrete design using pozzolans and optimization techniques. *Materials Today: Proceedings*, 63, 1351-1357.
17. Levers, C., & Bock, J. (2020). Cement production and global CO2 emissions: An overview. *Journal of Environmental Management*, 255, 109857.
18. Mehta, P. K., & Monteiro, P. J. M. (2014). *Concrete: Microstructure, Properties, and Materials*. McGraw-Hill.
19. Scheffe, H. (1958). Experiments with mixtures: Designs and analysis. *Journal of the American Statistical Association*, 53(282), 828-845.

20. Ranjbar, N., Saidi, M., & Fadaei, A. (2019). Predictive modeling of compressive strength in concrete using a hybrid approach. *Construction and Building Materials*, 210, 84-92.
21. Sharma, D., & Gupta, S. (2020). Valorization of agricultural waste in concrete: An overview. *Journal of Cleaner Production*, 259, 120771.
22. Simon, M. (2003). Polynomial regression models in statistical analysis. *Journal of Statistical Software*, 8(2), 1-22.
23. Singh, A., & Goyal, R. (2021). Optimization of concrete with pozzolans using Scheffe's method. *Materials Today: Proceedings*, 44, 123-128.
24. Thakur, P., Yadav, N., & Singh, A. (2021). Agricultural waste in concrete: Properties and durability. *Sustainable Materials and Technologies*, 29, e00254.
25. Yuan, H., Zhang, J., & Chen, S. (2018). Application of simplex method in concrete design. *Materials Science and Engineering: A*, 734, 15-23.
26. Zhang, L., et al. (2020). Mathematical modeling in concrete design: A review. *Materials and Structures*, 53(2), 1-13.

PUBLICATIONS

1. L. Anyaogu, Okere. C. E, J. I Arimanwa, **Iwuagwu, E. O**. (2022). Comparative analysis of the Dynamic Behaviour of Coupled and Non-Coupled Shear Walls in Tall Buildings**.** American Journal of Sciences and Engineering Research, https://www.iarjournals.com/Vol5,5.html.

2. **Iwuagwu, E. O**., L. Anyaogu,, & Onyeka, J. O. (2024). Prediction of Compressive Strength of Calcined Clay-Saw Dust Ash-Cement Concrete Using Scheffe's Simplex Method. Journal of Materials and Manufacturing Processes (Under Review).