



# Modeling of Granular Flow Velocities of Different Shapes by Neural Network Methods (GMDH, MLP) and Fuzzy Inferences (ANFIS)

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**ABSTRACT:** This study concerns the characterization and modeling of the average speeds of translation and rotation of aggregates of different shapes in flows of different viscosities of the immersion liquid. We made simulations with the ROCKY software by the discrete element method (DEM). The main parameters are the shape of the aggregates and the viscosity of the fluid. The flows are on an inclined plane of variable slope. The constitutive law is that of Newton.

The methods of artificial neural network (ANN) and the adaptive neuro-fuzzy inference system (ANFIS) were used for the analysis of the results obtained in order to carry out the modeling.

The ultimate goal of this work is to numerically describe the characteristics of flows according to the shape of granular materials and the viscosity of the immersion liquid by applying computer simulations. It was determined then that the experimental data can be estimated to a particularly close extent via the ANN and ANFIS models.

**Keywords:** Flow, DEM, granular materials, number of vertices, slope, viscosity, velocity, MLP, GMDH, ANFIS.

## I. INTRODUCTION

Aggregates, also known as granular materials, are composed of solid particles derived from rocks, soils, or broken materials. These granular substances typically have dimensions ranging from 1 to 100 mm and are obtained through processes such as crushing and screening. Aggregates are widely used in the manufacture of various construction materials, as well as in different layers of pavements. Their flow behavior on an inclined pipe is important in several industrial fields. [1], [2], [3], [4], [5].

The shape of grains has a significant impact on the mechanical behavior of granular materials. The behavior of an aggregate of grains is governed by the interaction between individual grains. The discrete element method is employed to effectively study the contact, separation, and frictional displacement between grains. Angular shaped elements are often used to accurately model the effects of grain shape.

In this study, the discrete element method was used to analyze the mechanical behavior of flow movements [6], [7]. The objective was to investigate the behavior of aggregates when immersed in a flowing medium on an inclined track (see Figure 1). The focus was primarily on the linear and angular velocities of the aggregates, as well as the interactions between them.

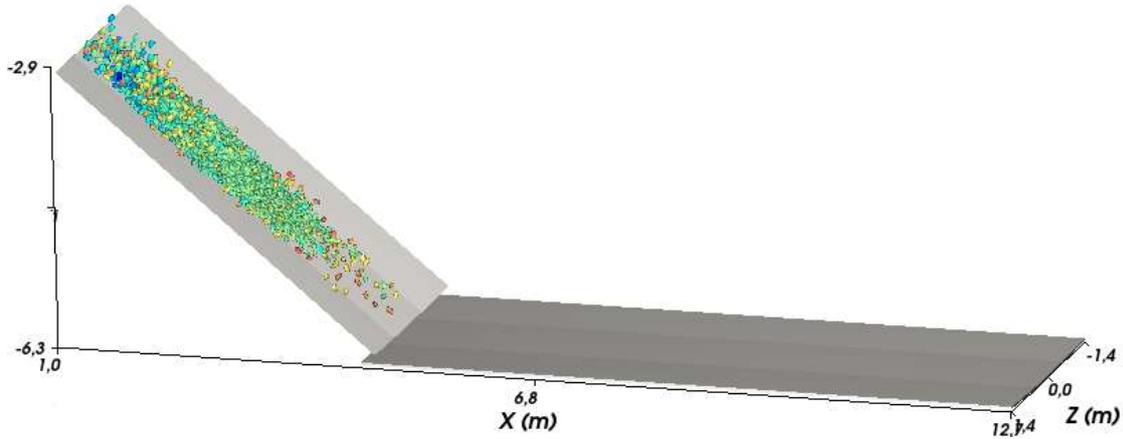
In the modeling stage, various models were employed to simulate granular flows, including the multilayer perceptron (MLP) artificial neural network, the Group Method of Data Handling (GMDH), and the Adaptive Neuro-Fuzzy Inference System (ANFIS). Finally, the results obtained from each model were compared

to determine which model was the most suitable for this type of flow.

**II. MATERIAL AND METHODS: dynamic study of granular flows**

**2.1. Description of the context and material**

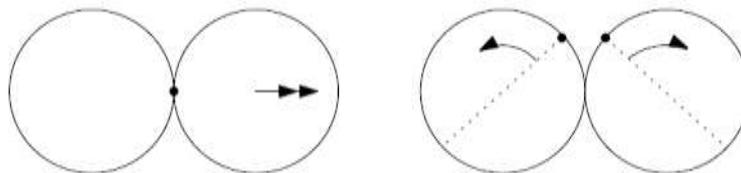
Simulations are carried out using the Rocky software to study the flows of aggregates immersed in a liquid on an inclined pipe.



**Figure 1 : Illustration of the simulation: experimental device (fluid flow is not shown)**

**2.2. The equations governing the flow**

During flow, each aggregate undergoes translational and rotational movements in three dimensions (3D) (Figure 2).



**Figure 2 : Translation and rotation between two aggregates in one dimension**

To visualize and predict the behavior of aggregates in the three-dimensional (3D) flow, we apply the discrete element method (DEM) on Newton’s laws. Each particle « i » has 6 degrees of freedom: 3 translations and 3 rotations.

The translation movements are governed by the following equation:

$$m_i \frac{d\vec{V}_i}{dt} = \sum_{j \neq i} \vec{F}_{ij} + \vec{F}_{surface} + \vec{F}_{hydro} + m_i \vec{g} \tag{1}$$

Where  $F_{ij}$  is the contact force exerted by particle  $j$  on particle  $i$ ,  $F_{surface}$  represents the force exerted by wall or wall on particle «  $i$  »,  $g$  is gravity and  $F_{hydro}$  are the hydrodynamic forces associated with the presence of a fluid applied to particle  $i$  (Archimedean thrust, drag, lift...).

We also write the law of evolution for the rotational contribution  $\omega_i$  from Newton’s second law:

$$J_{\Delta i} \frac{d\omega_i}{dt} = \sum_{j \neq i} M_{ij} + M_{surface} + M_{hydro} \tag{2}$$

Where  $M_{ij}$  is the moment of force exerted by particle  $j$  on particle  $i$  and  $M_{hydro}$  is the moment of torque exerted by the fluid on particle  $i$ . Note that weight does not contribute to the couple, as well as Archimedes' thrust.  $J_{\Delta i}$  is the moment of inertia of particle  $i$ . The external force  $F_{hydro}$  as well as the associated moment  $M_{hydro}$  are determined by the interaction between the fluid flow and the particle. These equations make it possible to know in detail the internal structure of the flow, that is to say for each solid in motion.

**2.3. Integration of Newton's equations: leapfrog method, Verlet's algorithm [5][8]**

To integrate Newton's equations of motion, we use Verlet's algorithm: calculate the grain trajectories step by step. Specifically, we follow the "leapfrog" step.

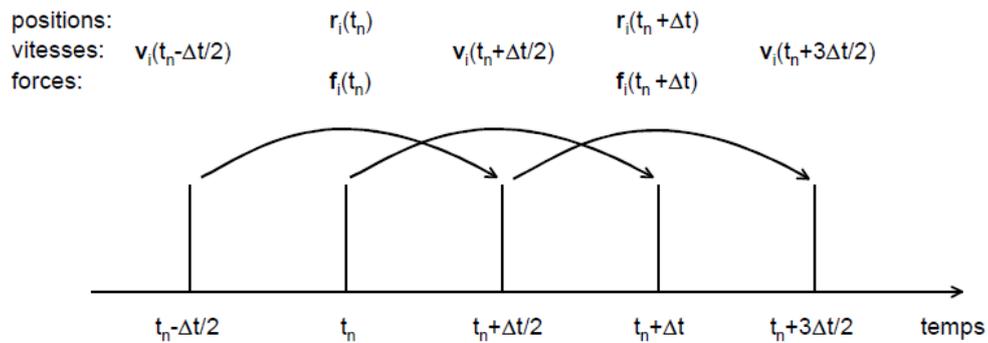


Figure 3 : Integrations of motion equations according to the leapfrog algorithm

**Summary of the algorithm :**

By knowing the positions, the velocities, the accelerations, the contact forces at time  $(t - \Delta t)$  as well as the velocities at time  $(t - \frac{\Delta t}{2})$ , we can calculate the positions, the velocities of each element at time  $t$  and the contact forces at this time by updating in case of contact . From this, we can calculate the accelerations at time  $t$ .

We continue this process step by step, calculating instantaneous velocities at time  $(t + \frac{\Delta t}{2})$  and continuing the resolution until the desired simulation duration is reached.

**III. HARDWARE : ROCKY DEM software**

We use the ROCKY DEM software to analyze the movements of each aggregate by modeling the contact forces between each neighboring particle.

**3.1. Aggregates**

The undeformable granular materials used in our study are quartz with a density of  $2660 \text{ Kg/m}^3$ . The size of the simulated granules is 0.1 m. Table 1 provides information on the the types of granules, while Table 2 gives the number of vertices, the mass, the moment of inertia, and the number of simulated granules for each type.

Table 1 : Type of aggregates

Type A	Type B	Type C	Type D	Type E

**Table 2 : Characteristics of 10mm size aggregates**

	Number of vertices	Mass (Kg)	Moment of inertia (Kg.m <sup>2</sup> )	Number of aggregates simulated
<b>Type A</b>	5	0.160	5.148 e-05	8840
<b>Type B</b>	10	0.575	5.067 e-04	2474
<b>Type C</b>	15	0.513	4.044 e-04	2534
<b>Type D</b>	20	0.561	4.664 e-04	2769
<b>Type E</b>	Spherical	0.714	4.575 e-04	1990

### 3.2. Study of the flow on an inclined slope

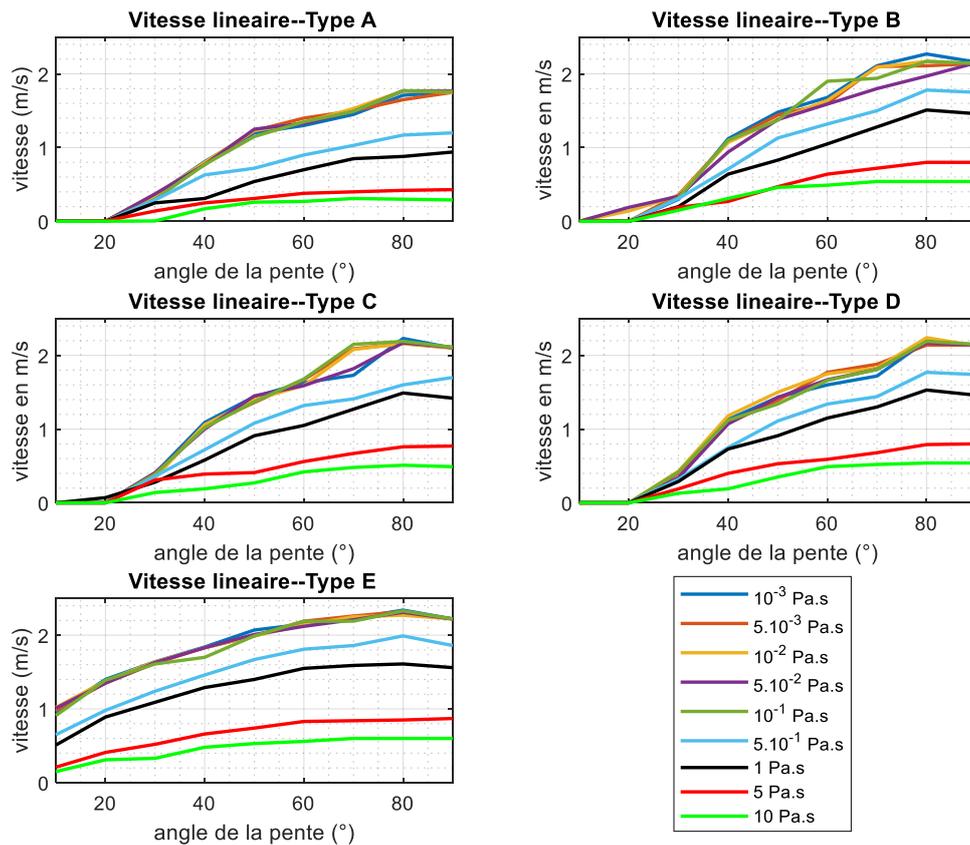
A flow of aggregates is deposited at the top of the inclined plane with a low flow of fluid. They move with the liquid under the effect of gravity  $g$  along the rectilinear trajectory of length  $L=5m$  and inclined at a variable angle  $\alpha$ . Flow tests are carried out at different slopes, for each type of aggregate and at different viscosities of the fluid. The different fluid viscosities expressed in Pascal-seconds are 0.001, 0.005, 0.01, 0.05, 0.1, 0.5, 1, 5, 10 (in Pa.s). The density of the liquid is fixed at  $1000 \text{ kg/m}^3$ . The angle of inclination of the inclined plane varies from  $10^\circ$  to  $90^\circ$ . The number of aggregates used during the simulation time varies according to the type of aggregates.

## IV. RESULTS

In this section, a series of simulations are described. They have were carried out to study the effects of the granule shape, liquid viscosity, and pipe inclination angle on the flow characteristics of the particles. The variation in linear velocity and angular velocity of the granules is then studied.

### 4.1. Linear velocity

On graph 4, we plotted the translational velocities of different types of granules (Type A, Type B, Type C, Type D, Type E) for different values of the immersing fluid viscosity and different angles of inclination of the pipe plane. The x-axis represents the inclination of the pipe in degrees. The colors of the curves indicate the exploited liquid viscosity.



**Figure 4 : Representation of the linear velocity profile of the flow.**

All the curves show that the flow velocity is significantly dependent on the inclination of the conduit. The higher the inclination angle, the higher the velocity. The observed velocity values range from 0 to 2.5 m/s. For non-spherical granular materials, a very low slope angle can block the flow, and there is a threshold angle below which the flow does not occur. This threshold is usually between 20 and 35 degrees.

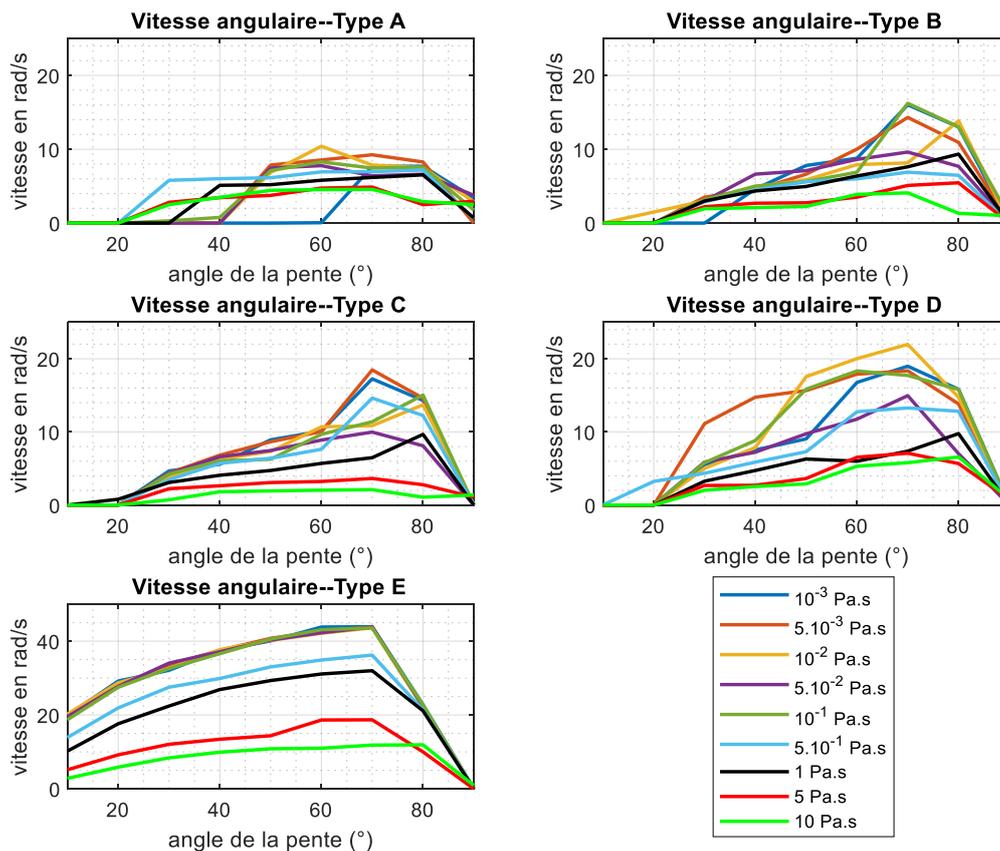
Regarding the shape of the granular material, for 5-cornered bodies (type A), the velocity is slightly lower compared to other types. However, for spherical granular materials, due to their spherical shape, they have higher velocities compared to other types of granular materials.

Granular materials of type B, type C, and type D have almost the same linear velocity profile. Their displacement velocities are the same for a given viscosity. In fact, the most commonly used granular shapes in practice are type B, C, or D.

Regarding the viscosity of the liquid, we observe that for all types of granular materials, some colors coincide. The effect of viscosity did not appear for values between 0.001 and 0.1 Pa.s. For fluids with viscosity higher than 0.5 Pa.s, the flow is more difficult. Viscosity causes friction on the displacement of the granular materials. However, in general, the effect of viscosity starts to become noticeable from this value of 0.5 Pa.s.

#### 4.2. Angular velocity

The rotation of the solids depends heavily on the slope and their sphericity, which is the moment of inertia of each granular material. These differences are represented in the following figures (Figure 5).



**Figure 5 : Profile of the angular velocity of the granular materials**

The rotation varies from 0 to 22 rad/s for the cornered granular materials (A, B, C, D), while spherical granular materials can reach a rotational speed of 45 rad/s.

For type B and type C granular materials, their angular velocities during flow are almost identical for each viscosity studied.

The effect of viscosity became noticeable starting from 0.5 Pa.s. From this value, the rotation of the granular materials decreased: green, red, black, and sky blue colors.

Regarding the inclination of the conduit, the rotational velocity varies according to the slope of the flow. For all types of granules, the maximum rotational velocity is obtained for an inclination of 70° or between 60° and 80°. However, it is between 45° and 55° that the rotation is stable for all types of granules, defining it as the important inclination. To maintain satisfactory rotational conditions, a moderate slope is necessary.

It is important to note that the rotational velocity of spherical granules is higher than that of other types of granules for all simulated viscosities. Thus, if one wants to use the rotation of granules, those with a spherical shape offer the maximum value. However, if the shape of granules cannot be chosen, it is recommended to choose an inclination between 45° and 65° to obtain adequate rotation of the bodies during the flow.

## V. ARTIFICIAL NEURAL NETWORK MODELING

We developed three models for predicting the flow properties of granular materials using 405 simulation data points generated using the ROCKY DEM software (Figure 5). The methods employed included artificial neural networks such as GMDH and MLP, as well as a neuro-fuzzy system ANFIS for predicting the rotational velocity of granular materials during flow.

We will compare and validate these methods using prediction tests and evaluate performance using metrics such as Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and correlation coefficient R.

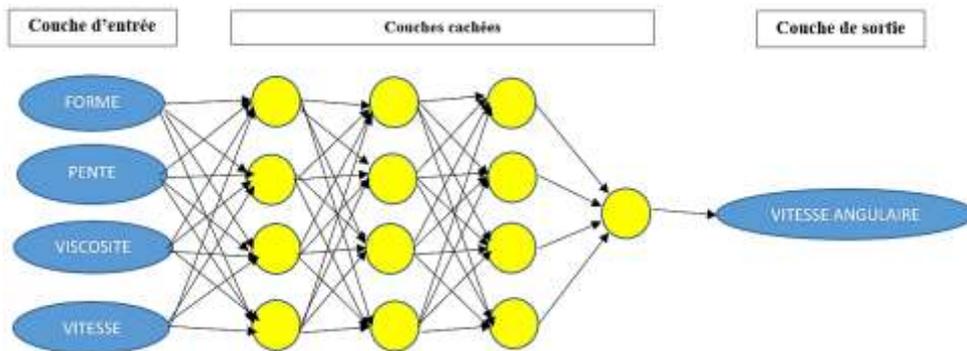
**5.1. Neural GMDH: Group Method Data Handling**

The Group Method of Data Handling (GMDH) is a family of mathematical modeling and nonlinear regression algorithms. This approach is also known as a polynomial neural network and can be considered as a specific type of supervised artificial neural network (ANN). In addition to modeling specifications, GMDH uses the concept of natural selection to control the size, complexity, and accuracy of the network. The main application of GMDH is modeling complex systems, function approximation, nonlinear regression, and pattern recognition..[9], [10]

**5.1.1. Model of Angular Velocity Proposed by GMDH :**

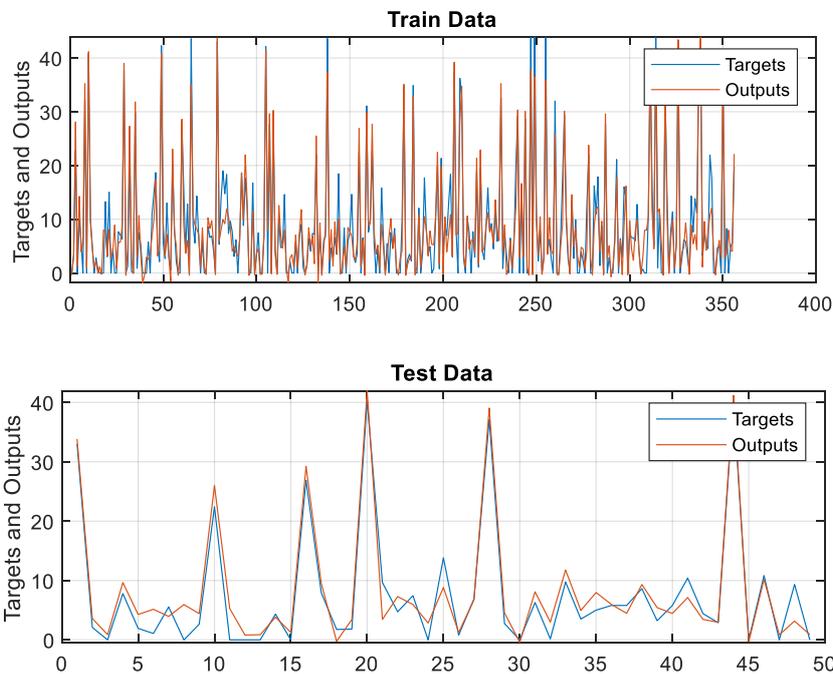
To predict the value of the rotational speed, which varies from 0 to 45 rad/s, we took into account the parameters of shapes, slopes, viscosities, and linear velocities to make this prediction. The data used were raw data that were randomly mixed to avoid any bias of order or group in the analysis.

We used the shape, slope, viscosity of the liquid, and linear velocity as inputs to predict the output rotational speed. The neural network thus formed consists of four hidden layers: the first three each contain 4 neurons, and the last layer contains one neuron. These parameters were chosen to optimize the accuracy of the predictions.



**Figure 6 : Architecture of the GMDH neural network for the considered model**

The data is usually divided into two distinct sets: a training phase that uses about 80% of the data to train the model or algorithm, and a testing phase that uses the remaining 20% to evaluate the model's performance. So, we obtained the following curves, which allow us to evaluate the quality of the model..



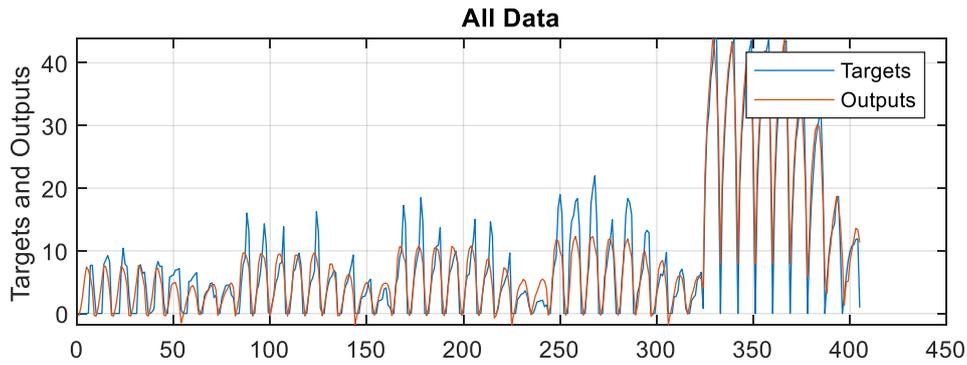


Figure 7 : Essai et test des données par GMDH

By analyzing the curves of the training phase and the testing phase, we were able to evaluate the quality of the model. The results obtained showed that the model was acceptable and capable of generalizing correctly on unknown data.

**5.2. Réseau de neurones artificiels de type MLP : multi layer perceptron**

We used a multi-layer perceptron neural network to model the rotational speed of the flow. The input parameters used for the model are shape, slope, viscosity, and translational velocity. As output, we have the rotational speed of the flow, which we sought to predict from the input data. The data used is still randomly mixed to avoid any bias of order or group in the analysis.

**Angular velocity model**

To model the rotation speed of the aggregates, five hidden layers are connected in the neural network (Figure 8), the first contains 40 neurons, the second and third each contain 30 neurons, the fourth contains 6 neurons, and the fifth layer contains 9 neurons. The neural network optimization parameters are then based on the following neural architecture:

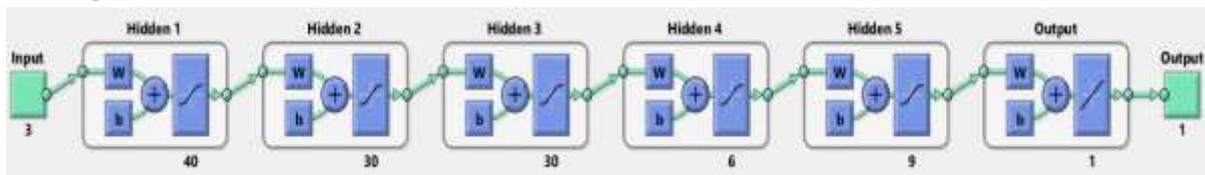
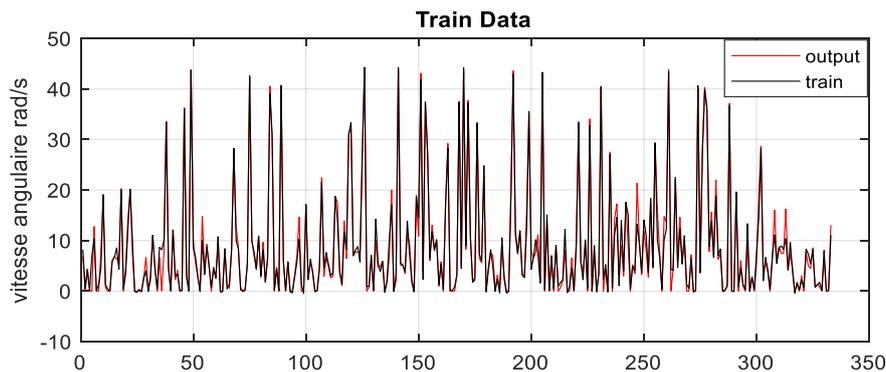


Figure 8: Neural architecture of the angular velocity model using MLP

The data is divided into two phases:

- Training phase: 80% of the data
- Testing phase: 20% of the data We then have the following training and validation results :



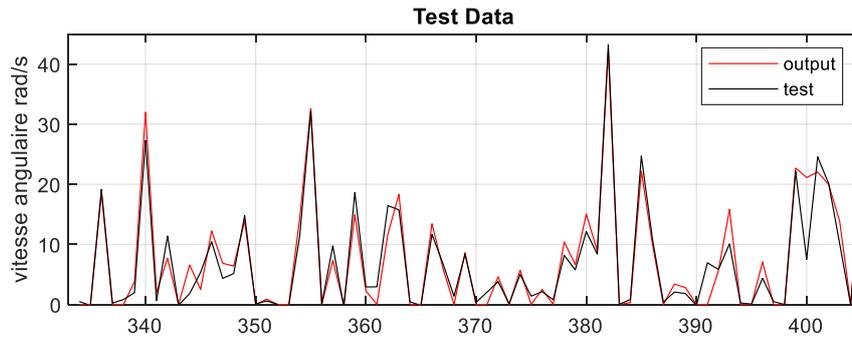


Figure 9 : Testing and validation of the data using MLP

In summary, evaluating the MLP model based on the curves from the training and testing phases has shown good results. This suggests that the model is reliable for practical use.

**5.3. ANFIS (Adaptive Neuro-Fuzzy Inference System)**

ANFIS is a new type of neural network that combines fuzzy logic and neural network. It is a type of multi-layered network that builds a fuzzy inference system (FIS) where the parameters of the membership function are optimized using a neural network. ANFIS provides a tool for fuzzy modeling of the data set by adjusting the parameters of the membership function to obtain the best model. The learning methods used in ANFIS are a backpropagation algorithm and a hybrid algorithm.

**Angular velocity model**

As with the two previous methods, we used form, slope, viscosity, and linear velocity as input parameters, and rotational speed as the output parameter (see Fig. 10). This means that we introduced 4 input parameters and one output. The data was permuted to improve analysis during testing.

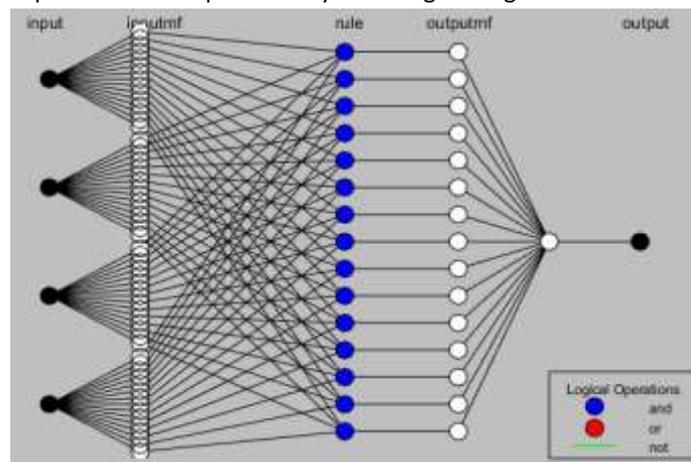


Figure 10 : Structure of the neuro-fuzzy system used

Each input parameter is utilized in 15 membership functions. These utilized parameters are summarized in Table 2.

Table 2 : The different layers of an ANFIS system

Layers	Types of layers	Number of neuron
Layer 0	Inputs	4
Layer 1	Fuzzification	4 × 15 = 60
Layer 2	Rules	15
Layer 3	Normalisation	15
Layer 4	Défuzzification	1
Layer 5	Output	1

We have the following curves for the training phase and testing phase in order to evaluate the quality of the

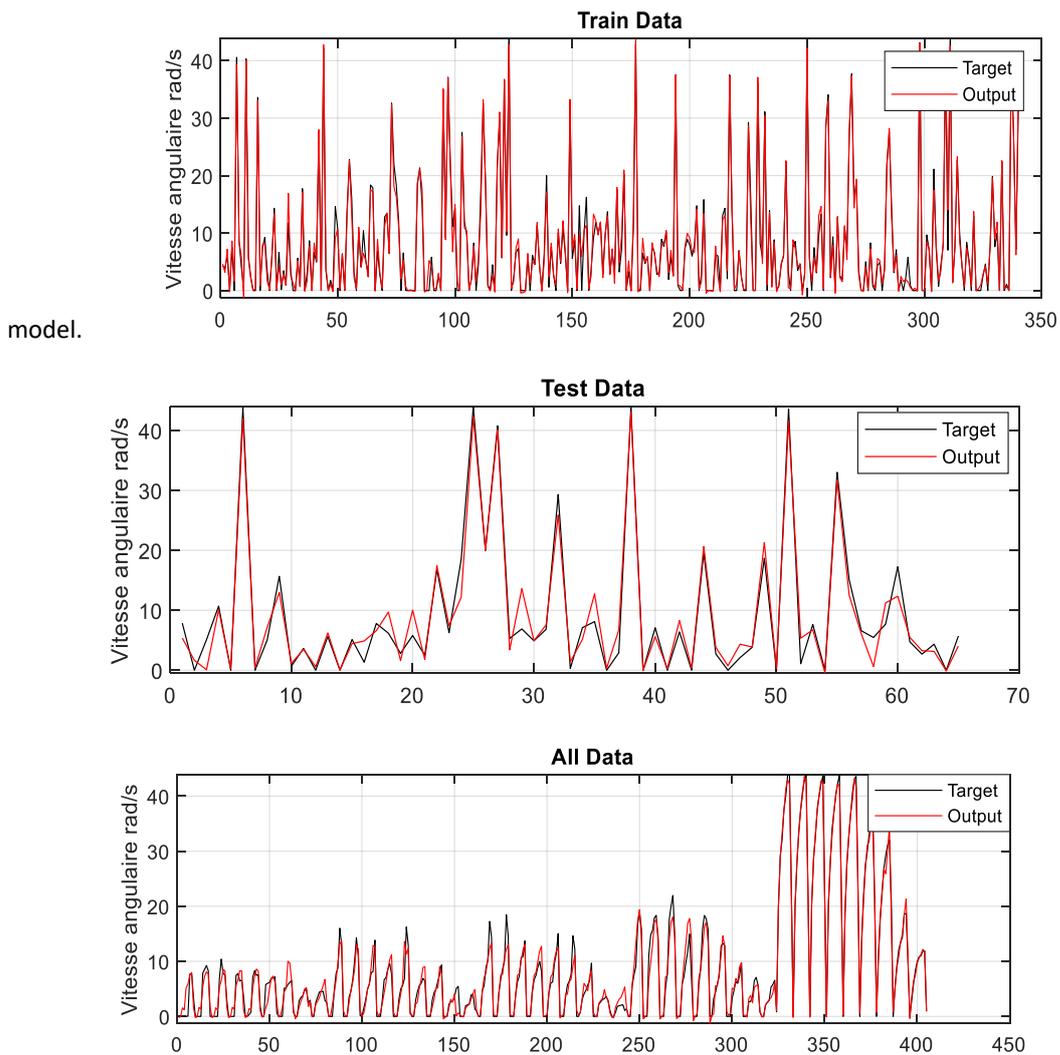


Figure 11 : Testing and validation of the data

## VI. COMPARISON OF THE MODELS OBTAINED AND DISCUSSION

3D modeling of granular flows is a rapidly growing research area with applications in various industrial sectors. 3D granular flows are often more complex and difficult to model than 2D flows [11] due to the three-dimensional nature of the particles. However, these flows are often closer to reality and can provide more accurate information on observed phenomena. In this study, we used 3D simulation via the ROCKY software to analyze linear and angular velocity of the particles and obtain more detailed results.

As a specific feature of this work on granular materials, we continued our study by using artificial intelligence to model the obtained results and evaluate the correlations between the model outputs and the simulation results. The results obtained with the MLP neural network and the neuro-fuzzy system are significantly superior (with the correlation coefficient  $R$  closer to 1) compared to the GMDH method. The differences between the three models are illustrated in the three graphs. However, while these differences are significant, all three models exhibit acceptable performance.

It should be noted that the neuro-fuzzy system proved to be the most effective method of the three for our case (see figure 12).

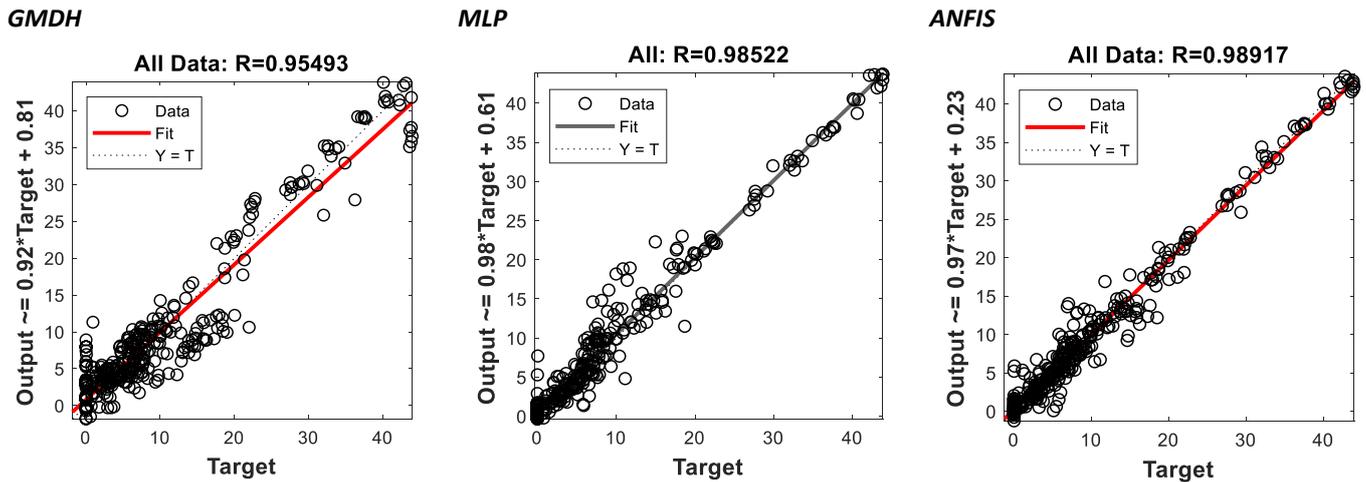


Figure 12 : Correlation between the model output and the output of angular velocity simulation results

The following table (Table 3) presents the prediction test and validation errors for the output “angular velocity of granular rotation” during flow, using the three previously described methods:

Tableau 3: Analyse statistique de l'estimation de la vitesse angulaire des écoulements calculée par les méthodes d'intelligence artificielle

METHODS	Train			Test			All data		
	R	MSE	RMSE	R	MSE	RMSE	R	MSE	RMSE
MLP	0.987	2.522	1.588	0.976	6.794	2.606	0.985	3.282	1.811
GMDH	0.952	10.754	3.279	0.974	6.285	2.507	0.955	10.213	3.195
ANFIS	0.991	1.904	1.380	0.980	5.590	2.364	0.989	2.496	1.580

In short, methodologies on artificial neural networks of GMDH and MLP type, as well as on the neuro-fuzzy system, were used to process granular flow data. Although the tests and predictions are almost identical, it can be observed that the neuro-fuzzy system has the lowest errors, both for the prediction tests and validation.

### VII. CONCLUSION

In summary, the DEM calculation method enables the description of individual movements of each granule in their flow, using numerous numerical simulations to describe their behaviors such as linear velocities and rotational velocities. The simulation results showed that the linear flow velocity is significantly influenced by the inclination of the conduit, with an increase in velocity observed as the inclination angle increases, ranging from 0 to 2.5 m/s. Regarding angular velocity, corner granules (type A, B, C, D) can reach a maximum rotational speed of 22 rad/s, while spherical granules can reach 45 rad/s, with smoother flows.

After conducting numerical simulations of the flows, we applied artificial intelligence modeling to these flows. To evaluate each model, we calculated the difference between the results from the ANFIS, MLP, GMDH models, and raw data (before modeling). RMSE and MSE were the metrics used. According to our experiments, the neuro-fuzzy ANFIS modeling is the best, as it corresponds to the minimum of RMSE and MSE. This suggests that the neuro-fuzzy model is well suited to the learning task in question and can be used for practical applications in the field of granular flow.

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