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Second Order Drilling Rate of Penetration Model Using Operating Parameters and Step-wise Linear Regression Model

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Abstract: *Drilling, the making of a conduit to reach economic reserves of petroleum resources for production is a cost consuming process. The cost involved during drilling is such that the duration of the drilling directly influences and dictates the cost of drilling. This is because a major fraction of the drilling cost are computed based on the hour spent during the drilling activities, therefore a reduction in the drilling cost will be achieved by reducing the number of days spent during the drilling process. This we can achieve through observation of series of standards and practices designed to reduce non productivity and improve the drilling efficiencies, generally termed drilling optimization. One way of optimizing drilling is ensuring that the rate at which footage is gained (ROP) is not too slow nor too fast but should be at the optimum speed. ROP optimization as it is widely called involves series of procedures designed to improve the accuracy in prediction of the rate of penetration and also determine the optimum rate of penetration settings that should be obtained to achieve non-productive time reduction consequently cost reductions. One of the methods used in estimating the rate of penetration as a function of the controlling variables is to use a linear regression model, with the rate of penetration as the dependent variable whose value depends on the values of the other variables in the model. The advantage of the regression model is that the ROP can be optimized using the regression model by directly altering each of the variables in the model to see the response in the ROP thereby determine the optimum setting or the model can be optimized using mathematical techniques. In this project, a nonlinear model is developed to predict the rate of penetration as a direct function of controllable operating parameters, namely the hydraulic weight applied per inch of bit, WOB and the rotation speed in revolution per minute. Optimum ROP is then determined by minimization using partial derivatives of the ROP model thus generated.*

Keywords: *rate of penetration; ROP optimization; rotary speed; weight on bit; nonlinear regression.*

I. Introduction

The rate at which the drill bit advances through the rocks under it, advancing into the formation is generally referred to as the rate of penetration. The ROP is helpful in nonproductive time minimization consequently cost minimization. This is because the speed at which a conduit is safely drilled will determine the number of days that will be spent on the drilling and since drilling is a cost per hour operation, the cost will be reduced if the number of days consequently hours is significantly reduced. Several literature have been released with the contributions of various authors and researchers on the subject matter of efficient ROP prediction and optimization. There have been proposed empirical models, notable among which are the models of Bourgoyne and Young, popular as the widely used model in the oil industry and referred to as BYM. The BYM employed a mathematical relationship modelling of ROP using multiple linear regression as a

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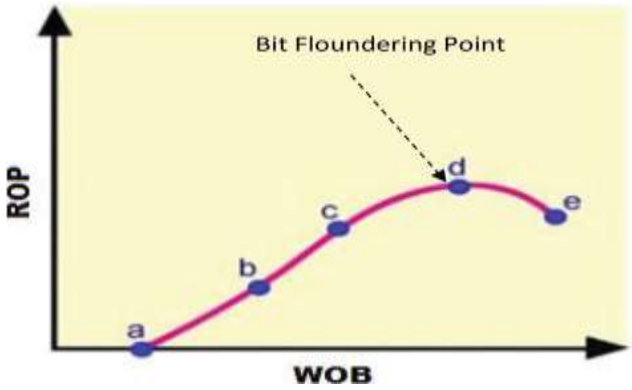
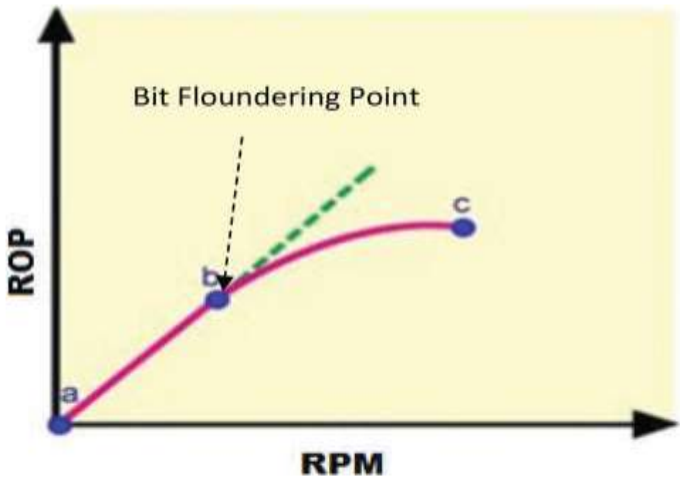
function of eight different parameters namely, the formation strength, normal compaction characteristics, under compaction exponents, hydraulic exponent, pressure differential exponent, bit weight, rotary speed and tooth wear exponents.

Galle and Woods investigated and presented procedures for the determination of the best combinations of constant effective weight on bit and the rotation speed in revolutions per minute that will predict and yield the optimal value for ROP. They considered the effects of bit cutting dullness, effective weight per inch of bit and the rotation per minute on the rate of penetration.

Maurer also came up with an equation for estimating the ROP as a function of parameters like the weight on bit, effective bit size, rotation per minute, and the strength or hardness of rock based on perfect hole cleaning conditions.



The graph shown above is a schematic of the ROP against the effective weight applied per inch of the drilling bit based on recommended standards in line with the results from past literature.
As seen from the graph that the relationship between the ROP and the effective WOB increases linearly from point a up to point c, reaching point d where it attains a maximum value and afterwards the trend decreases as the ROP decreases with further increase in the WOB. This kind of trend is characteristic of a non-linear relationship possessing a maximum value and a turning point.



Similarly, a plot of the ROP against the rotation speed in revolutions per minutes also reveal a linear trend that indicates the ROP increases with increasing rotation speed up to point b, but as rotation speed continues to increase beyond point b, the ROP changes direction and begins to decrease, due to what is regarded as “bit floundering” This again indicates a nonlinear relationship between the ROP and the rotation speed, N.

Therefore, in this project, a nonlinear model of the ROP as a function of these two operating parameters is developed for estimating the rate of penetration and the resulting model is afterward used in optimizing the ROP and improving the overall drilling performance.

II. Materials and Methods

A nonlinear model precisely a quadratic model for predicting the rate of penetration as a function of the effective weight applied on the bit and the rotation speed is generated.

Assuming Zero foot will be gained per hour, at net effective zero weight applied on the drilling bit with no rotation, the equation of the model is written as:

$$\mathrm{ROP} = \mathrm{b_0WOB} + \mathrm{b_1RPM} + \mathrm{b_2WOB^2} + \mathrm{b_3RPM^2} \qquad (1)$$

Where ROP = Rate of Penetration [ft/hr], WOB = Effective Weight on bit [lbf/in], RPM = Rotation speed [revolutions per minute] and b_0 - b_3 as constants. In using the equation the values of the unknown constants has to be determined.

The constant was determined using nonlinear least squares method.

Having four unknown constants, we shall require a minimum of four equations to get a particular solution for the constants. This is achieved by generating the normal equations.
Generating the normal equations, you obtain the sets of equation below:

$$\sum ROP = \mathrm{b_0}\sum WOB + \mathrm{b_1}\sum RPM + \mathrm{b_2}\sum \mathrm{WOB^2} + \mathrm{b_3}\sum \mathrm{RPM^2} \qquad (2)$$

$$\sum ROP.WOB = \mathrm{b_0}\sum \mathrm{WOB^2} + \mathrm{b_1}\sum RPM.WOB + \mathrm{b_2}\sum \mathrm{WOB^3} + \mathrm{b_3}\sum \mathrm{WOB.RPM^2} \qquad (3)$$

$$\sum ROP.RPM = \mathrm{b_0}\sum RPM.WOB + \mathrm{b_1}\sum \mathrm{RPM^2} + \mathrm{b_2}\sum \mathrm{WOB^2.RPM} + \mathrm{b_3}\sum \mathrm{RPM^3} \qquad (4)$$

$$\sum ROP.\mathrm{WOB^2} = \mathrm{b_0}\sum \mathrm{WOB^3} + \mathrm{b_1}\sum RPM.\mathrm{WOB^2} + \mathrm{b_2}\sum \mathrm{WOB^4} + \mathrm{b_3}\sum \mathrm{RPM^2 \cdot WOB^2} \qquad (5)$$

Translating these equations in matrix format, the following equation is obtained:
The General Format: b.x = C
Where b = vector of the unknown constant.
X = matrix of the operating parameters.
C = vector of the constant on the other side of the equation.

Therefore

$$\mathrm{b} = \begin{bmatrix} \mathrm{b_0} \\ \mathrm{b_1} \\ \mathrm{b_2} \\ \mathrm{b_3} \end{bmatrix};\ \mathrm{x} = \begin{bmatrix} \mathrm{WOB} & \mathrm{RPM} & \mathrm{WOB^2} & \mathrm{RPM^2} \\ \mathrm{WOB^2} & \mathrm{RPM.WOB} & \mathrm{WOB^3} & \mathrm{WOB.RPM^2} \\ \mathrm{RPM.WOB} & \mathrm{RPM^2} & \mathrm{WOB^2.RPM} & \mathrm{RPM^3} \\ \mathrm{WOB^3} & \mathrm{WOB^2.RPM} & \mathrm{WOB^4} & \mathrm{RPM^2WOB^2} \end{bmatrix};\ \mathrm{C} = \begin{bmatrix} \sum ROP \\ \sum ROP.WOB \\ \sum ROP.RPM \\ \sum ROP.\mathrm{WOB^2} \end{bmatrix}$$

To obtain the numerical value for the sums, the table of values is prepared using the data obtained. The data consists of over 40 data points of real time data, obtained from a well within depths 7000ft to 12000ft, the longest interval section at an interval of 200ft.

The data is cleansed of irregularities and void of other parameters not necessary in this modelling.
This data is then imported into a statistical tool developed to aid the computation of the nonlinear model. The resulting output is displayed below

Multiple Nonlinear Regressor

File Data Graphs

TVD[ft]	ROP.WOB	ROP.RPM	ROP.WOB2	ROP [ft/hr]	WOB [lbf/in]	RPM	WOB^2	WOB^3	WOB^4
6000	1312.98	9480	36369.546	47.4	27.7	200	767.29	21253.9	588733.94
6200	1899.16	12020	60013.456	60.1	31.6	200	998.56	27660.1	766185.10
6400	818.44	6383.2	21197.596	31.6	25.9	202	670.81	18581.4	514705.80
6600	757.33	7014.9	16434.061	34.9	21.7	201	470.89	13043.7	361309.18
6800	1096.12	8261.8	29376.016	40.9	26.8	202	718.24	19895.2	551098.36
7000	1316.92	5953.2	38454.064	45.1	29.2	132	852.64	23618.1	654222.14
7200	2114.47	7787	74640.791	59.9	35.3	130	1246.09	34516.7	956112.39
7400	1938.3	5733	68809.65	54.6	35.5	105	1260.25	34908.9	966977.22
7600	2059.68	10604.9	69205.248	61.3	33.6	173	1128.96	31272.2	866239.71
7800	2124.4	7288.5	79877.44	56.5	37.6	129	1413.76	39161.2	1084763.
8000	3003.95	9711.4	109644.175	82.3	36.5	118	1332.25	36903.3	1022222.1
8200	1236.15	3450.5	45613.935	33.5	36.9	103	1361.61	37716.6	1044749.7
8400	1185.346	3196.09	45280.2172	31.03	38.2	103	1459.24	40420.9	1119660.
8600	3183.84	11256	126080.064	80.4	39.6	140	1568.16	43438	1203233.4
8800	3031.56	10827	114592.968	80.2	37.8	135	1428.84	39578.9	1096334.6
9000	3413.34	13494.6	132096.258	88.2	38.7	153	1497.69	41486	1149162.
9200	2828.52	12101.4	109746.576	72.9	38.8	166	1505.44	41700.7	1155109.0
9400	3295.74	12390	131500.026	82.6	39.9	150	1592.01	44098.7	1221533.3
9600	1740.99	7752.5	68420.907	44.3	39.3	175	1544.49	42782.4	1185071.7
9800	1767.48	6220.5	72820.176	42.9	41.2	145	1697.44	47019.1	1302428.7
10000	1785.29	5883.8	79088.347	40.3	44.3	146	1962.49	54361	1505798.9
Total	**41910.006**	**176810.29**	**1529261.52**	**1170.93**	**736.1**	**3208**	**26477.2**	**733417**	**20315652.**

Figure 1. Multiple nonlinear regression tool displaying data and the sums

Substituting the values for the sums as obtained from the statistical tool, we get the following:

$$\mathrm{x} = \begin{bmatrix} 736.1 & 3208 & 26477.2 & 513326 \\ 26477.2 & 110056.7 & 733417 & 17184096.1 \\ 110056.7 & 513326 & 3893936.5 & 85668286 \\ 733417 & 3893936.5 & 20315652.5 & 595899689.3 \end{bmatrix}; \; \mathrm{C} = \begin{bmatrix} 1170.93 \\ 41910 \\ 176810.2 \\ 1529261.5 \end{bmatrix}$$

Solving the equations, we obtain the following values for the unknown constants.

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0.018 \\ 0.884 \\ -0.0021 \\ -0.0032 \end{bmatrix}$$

Therefore, substituting the values for the constant in equation 2 above, the nonlinear ROP model will be written as

$$\text{ROP} = 0.018\text{WOB} + 0.884\text{RPM} - 0.0021\text{WOB}^2 - 0.0032\text{RPM}^2 \qquad (6)$$

III. Optimization

The model developed above is subject to optimization, the optimization is simply determining the maximum value of effective weight on bit and rotation speed for which the ROP has the maximal value. Since this is a nonlinear model we can use the partial derivative method to obtain the points of local maxima.

To do that we obtain the partial derivatives of the ROP with respect to the WOB and the RPM and set them separately to zero. This is because at the maximum point, the rate at which the ROP changes with respect to each of the variable should be zero. The resulting Partial derivatives are written below.

$$\text{ROP'}_{\text{RPM}} = 0.884 - 0.0064\text{RPM} \qquad (7)$$

$$\text{ROP'}_{\text{WOB}} = 0.018 - 0.0042\text{WOB} \qquad (8)$$

The resulting equations is then solved, setting the ROP to Zero and the resulting values for the RPM and WOB as shown below:

$$0.884 = 0.0064\text{RPM}$$

Therefore:

$$0.884/0.0064 = \text{RPM}$$

$$\text{RPM} = 138 \text{ rev/min}$$

Similarly:

$$0.018 = 0.0042\text{WOB}$$

$$\text{WOB} = 0.018/0.0042$$

$$\text{WOB} = 4.28 \text{ 1000lbf/in}$$

IV. Results and Discussion

The ROP predictions from this model was compared with the actual values and the result is presented in a tabular and graphical output below. The error in the model ROP predictions was also computed alongside in the table.

Table 1. Error in model's prediction relative to the actual ROP

Actual ROP	Model Predicted ROP	Error In Prediction
59.9	60.2	-0.5%
54.6	54.5	0.2%
61.3	61.1	0.3%
56.5	56.9	-0.7%
82.3	81.9	0.5%
40.9	41.1	-0.5%

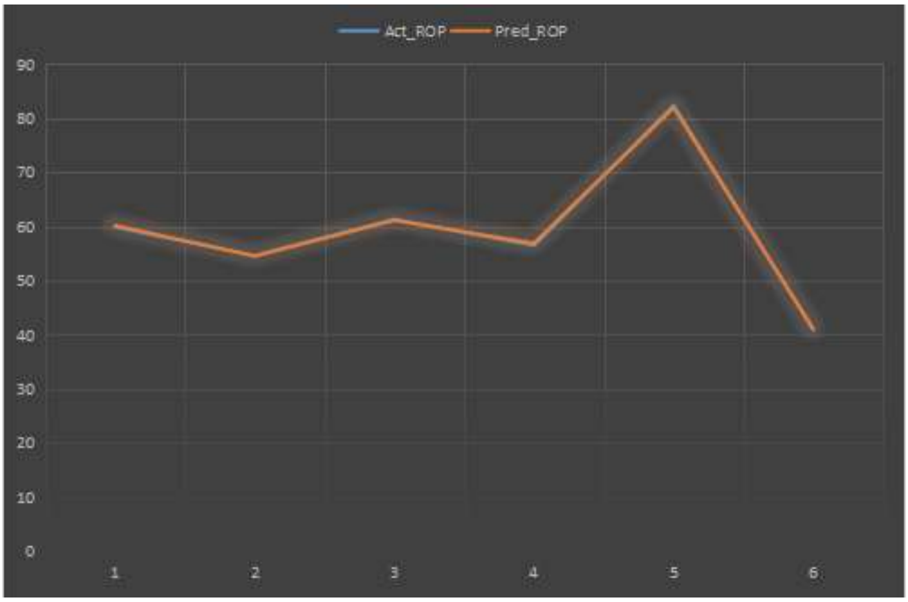


Figure 2. Actual ROP plotted with the ROP prediction from the model

As indicated by the trend markers, in the chart displayed above, generated with the statistical tool, the blue line indicates the actual ROP with the orange colored trend dictating the model's prediction and the results indicates the model prediction is overlapping that of the actual trend and this indicate that the model and actual values are quite similar.

Furthermore, the models prediction was compared against that of the bourgoyne and young's model with reference to the actual values, the result is displayed on a graphical plot below.

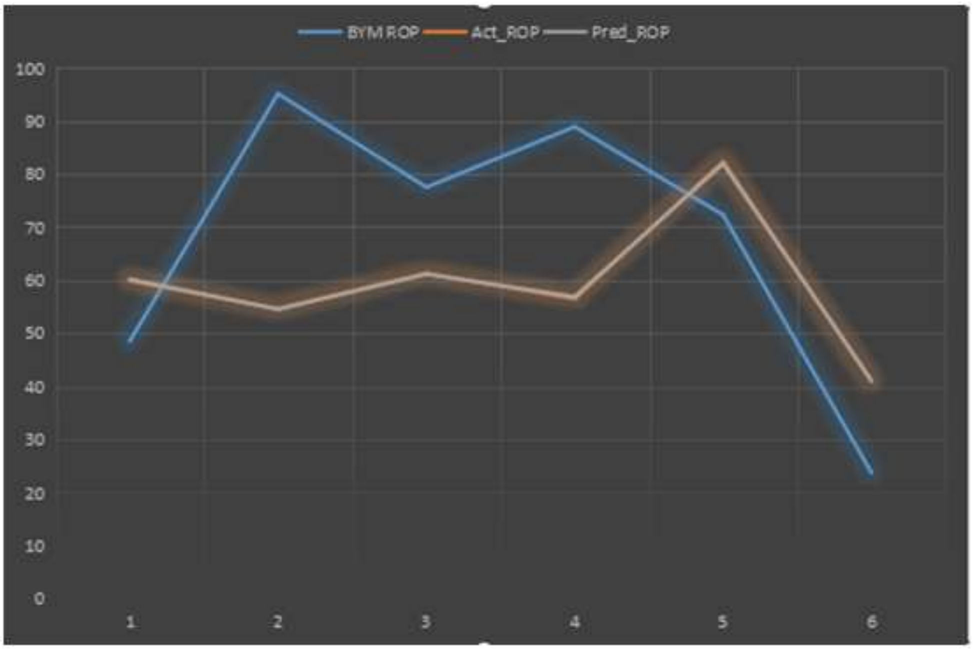


Figure 3. Model prediction against BYM against Actual values

In the plot above, the blue colored trend line indicates the BYM ROP predictions while the orange colored trend dictates the actual value of the ROP as read from the field data and the nearly gray colored trend line indicates the prediction of the model. The result reveals that the model has the greatest fit and shows the least variability in predicting and fitting the actual values of the ROP. This is because the ROP exhibits a nonlinear trend with the WOB and BYM as agreed by previous literature and the model adopts a nonlinear trend whereas the BYM adopted a multiple linear trend.

From the Optimization result, using the value of 4.28 thousand pounds force as the effective weight applied on the bit with a rotation speed of 138 revolutions per minutes, the optimal rate of penetration is obtained to be 180 ft/hr. Drilling at these rate would achieve a reduction of the number of days spent drilling by at least 5 days saving at least

V. Conclusions

The model performance agrees with the result from previous literatures that established a non-linear trend between the rate of penetration and the effective weight on bit and the rotation speed.
The model has a correlation coefficient of 0.98 with a maximum absolute error of 1%, this makes the model amenable to being used in ROP prediction and optimization.
Furthermore, the procedures used in this model can be applied in any geography, however suitable corrections must be taken into considerations using data from the selected geography.

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