



Study of Intercorrelation between Rainfall, Maximum Temperature and Sea Surface Temperature (SST)

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Abstract: The study area is the BOENY region, delimited in latitude between 15° south and 18° south and in longitude between 44° East and 48° East.

We studied the date of rupture of rainfall and maximum temperature by the Pettitt test then the intercorrelation between rainfall and maximum temperature and the intercorrelation between maximum temperature and sea surface temperature (SST) of the year 1979 to 2018. The result for the Pettitt rupture test, we have the year 2005 for the rainfall and the year 1997 for the maximum temperature. The study leads us that the data are intercorrelated with each other. According to this method, the maximum temperature is 251 days ahead of the rain and the maximum temperature is 3 days ahead of the SST.

Keywords: Rainfall, maximum temperature, sea surface temperature (SST), intercorrelation, Boeny region.

I. Introduction

Intercorrelation between data is a method used to extract the rate of resemblance between two signals, both one-dimensional, such as temporal signals, and two-dimensional. The goal is to find the delay (or translation t) that maximizes the similarity between the two signals [1].

The intercorrelation between $x(n)$ and $y(n)$ reaches a maximum for a lag k if $x(n)=y(n-k)$ [2]. When the maximum of the intercorrelation is reached, it means that the two signals resemble each other with a time lag equal to k .

The data used were averaged in space in order to obtain a vector representing the study area.

II. Materials and methods

2.1 Presentation of the study area

The study area (see Figure 1) is between latitude 15°South and 18°South and longitude 44°East and 48°East.

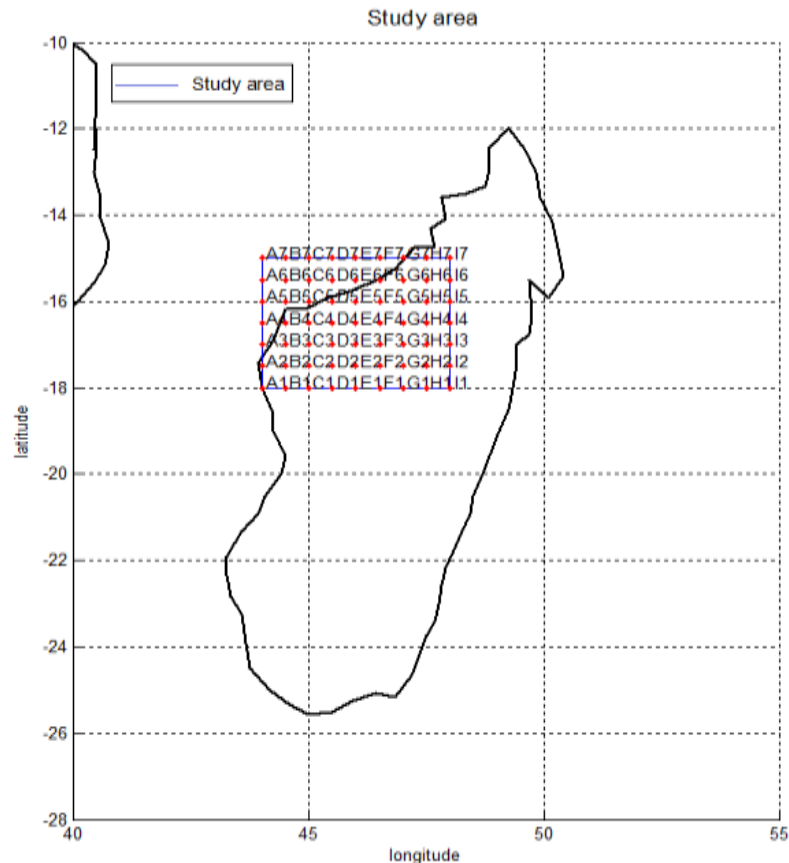


Figure 1: Study area $44^{\circ} \leq \text{longitude} \leq 48^{\circ}$ and $-18^{\circ} \leq \text{latitude} \leq -15^{\circ}$.

2.2. Databases

The meteorological data we used were derived from the daily experience re-analysis (ERA5) data of the ECMWF (European Center for Medium range Weather Forecasts) on a synoptic scale with a grid of $0.5^{\circ} \times 0.5^{\circ}$ of rainfall, maximum temperature and sea surface temperature over a time depth covering the period 1979-2018.

2.3 Rupture test

2.3.1 Test of Pettitt

The Pettitt test is dedicated to the detection of a rupture. The principle of this nonparametric test is to assess whether the pairs of values tend to be arranged preferentially in one direction, the pairs being formed by selecting a value on each side of a hypothetical breakpoint k (k is an integer between 1 and $n-1$). Pettitt's statistical test $U(k)$ is of the form:

$$U(k) = \sum_{i=1}^k [\sum_{j=k+1}^n \text{sgn}(x_j - x_i)] \quad (2.1)$$

where $\text{sgn}(x) = 1$ pour $x > 0$

$$\text{sgn}(x) = 0 \text{ pour } x = 0 \quad (2.2)$$

$$\text{sgn}(x) = -1 \text{ pour } x < 0$$

The Z statistic for this test is: $Z = \max |U(k)|$ avec $k = 1, 2, 3, \dots, n$ (2.3)

Let α , representing a risk of the first kind;

H_0 is rejected if a certain probability calculated from a studied time series is less than α [3] [4] [5].

The distribution function of Z can be approximated asymptotically by:

$$F(z) = P(Z > z) = 2e^{\frac{-6z^2}{n^3 + n^2}} \quad (2.4)$$

The Pettitt test also makes it possible to estimate the breaking point $\hat{k}$ such that:

$$\hat{k} = \text{Arg max } |U(k)| \quad (2.5)$$

with $k = 1, 2, 3, \dots, n$

The significance level is given by the following quantity:

$$Q = \pm \sqrt{\frac{-(n^3+n^2)\log(\frac{\alpha}{2})}{6}} \quad (2.6)$$

(Source: Partenariat 2010 Domaine : Savoirs Action n°8 Janv. 2011 - p 45/166)

$$H_0 \text{ is rejected if } F(z) = P(172 > z) = 2e^{\frac{-6z^2}{n^3+n^2}} < \alpha \quad (2.7)$$

$$\text{from where } z \text{ must check both conditions } z > \sqrt{\frac{-(n^3+n^2)\log(\frac{\alpha}{2})}{6}} \text{ et } z < 172 \quad (2.8)$$

Also, α must be less than 2, because $\log(\frac{\alpha}{2})$ should be less than zero. These conditions will not be fulfilled until $\alpha > 1.4$.

Conclusion: If we reject the H_0 hypothesis, there will be a risk of decision error of around 15%. (That's too high!).

2.4 Intercorrelation

2.4.1 Some Basics

The concept we propose to implement in this work is the intercorrelation between two signals. This concept aims to extract the resemblance rate between two one-dimensional signals (time signals for example). The mathematical definition of this concept, which we will try to intuit later, is that the intercorrelation between two signals, assumed to be one-dimensional in a first stage, of zero mean, $s(t)$ and $s'(t)$ is [6] [7] [8]:

$$\text{xcorr}(s, s')(t) = \sum_{i=-N}^N s_i s'_{i-t} \quad (2.9)$$

The principle of intercorrelation consists in searching, in segments of two data of the same dimension, the translation vectors which make it possible to find the maximum similarities between the data. In the formalism proposed in the preceding expression, it is a question of finding the delay t for which the intercorrelation $\text{xcorr}(t)$ is maximum.

Indeed, it is only for a delay t which maximizes the resemblance that the sum is effected in a coherent manner with products s_i and s'_{i-t} all of which are positive (in the order of s_i^2). Otherwise, the product of the values of s_i is produced with a signal s' of zero mean value which is most likely to give a result close to 0. Thus, we will search for t which maximizes the sum of the elements of the product $s(i)$ and $s'(i-t)$ when the delay t allows s to resemble at best s' offset by t .

III. Results and Discussion

3.1 Graphical representation of the model

Figures 2 and 3 show us the break tests for rainfall and maximum temperature data.

Figure 2 shows us that the numerical test by Pettitt's method allows us to affirm the existence of a break date within the rainfall data, the date is the year 2005.

For Figure 3, the date of the maximum temperature break is the year 1997.

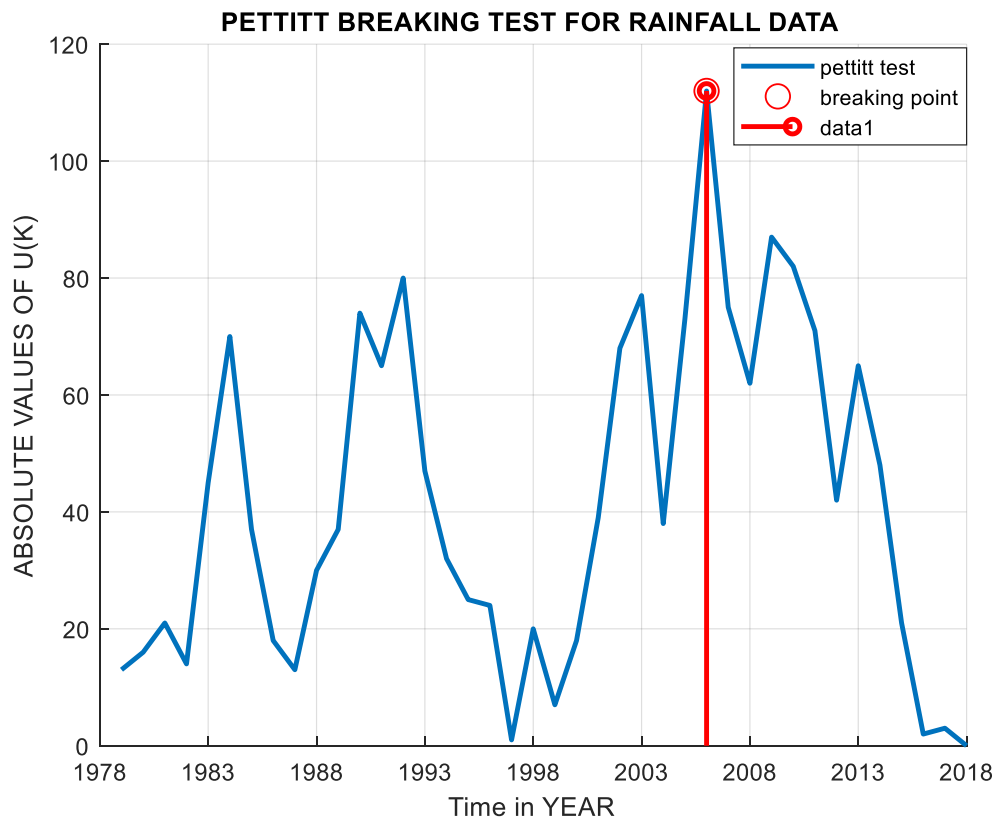


Figure 2: Rainfall rupture test curve

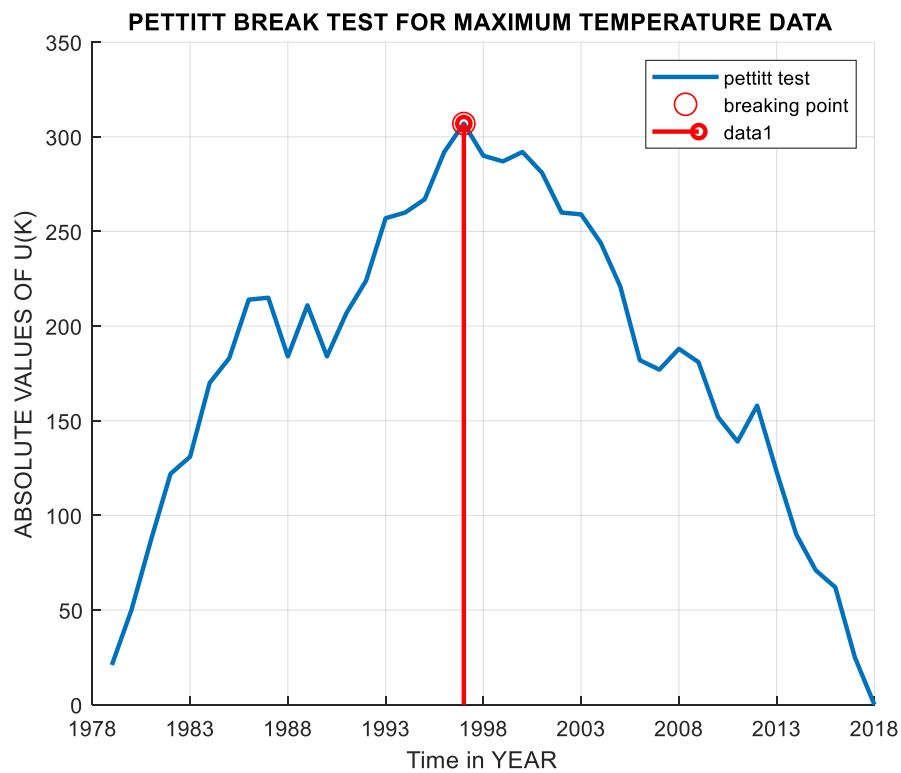


Figure 3: Maximum temperature rupture test curve

Figure 4 shows the result of the intercorrelation between rainfall and maximum temperature.

On this curve (Figure 4), the maximum value is 251. The temperature is 251 days ahead of the rain.

The formation of the rains is therefore due to the evaporation of the sea for 251 days, or about eight months and eleven days. With the value of the intercorrelation coefficient equal to 0.6395, we could say that the temperature can be correlated with the rain.

Figure 5 shows us the result of the intercorrelation between the sea surface temperature (SST) and the maximum temperature.

On this curve (figure 5), the maximum value is 3. So, we can interpret that the maximum temperature influences the sea surface temperature 3 days before. In other words, the maximum temperature is 3 days ahead of the SST.

The warming of the sea is therefore due to the radiation of the maximum temperature for 3 days. With the value of the intercorrelation coefficient equal to 0.8058, one could say that the SST is correlated with the maximum temperature.

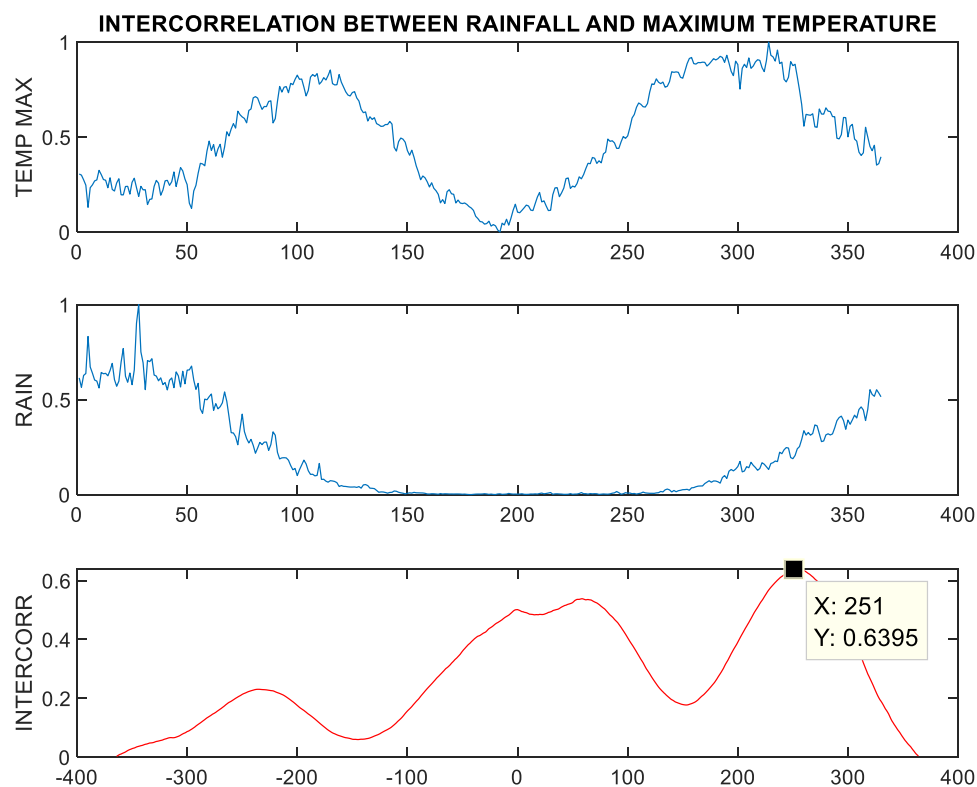


Figure 4: Intercorrelation curve of rainfall and maximum temperature

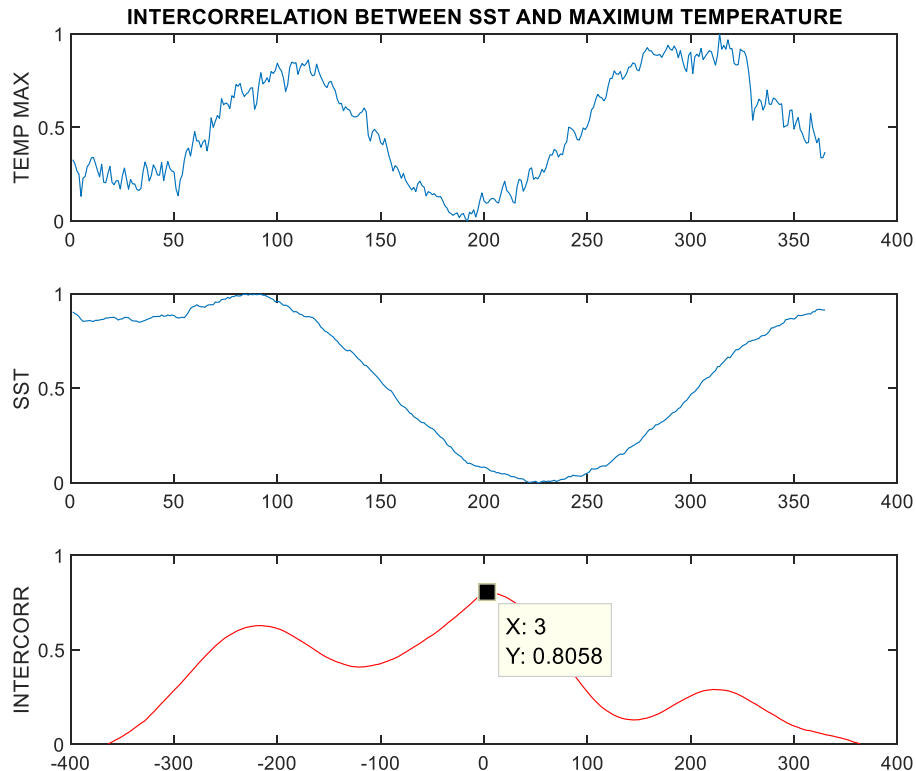


Figure 5: Intercorrelation curve of SST and maximum temperature

IV. Conclusion

In this article, we are interested in the quantitative analysis of rainfall, maximum temperature and sea surface temperature from 1979 to 2018 in the Boeny region of Madagascar. This part is located between longitude 44° East and 48° East, latitude 18° South and 15° South. To study the predictability of these parameters, it is necessary to make a quantitative study of some climatological parameters. In our case, we proceeded to the use of statistical methods, the Pettitt break test and the intercorrelation method.

According to the Pettitt rupture test, the rupture date is the year 2005 for the rainfall and for the case of the maximum temperature, the rupture date is the year 1997.

And the second method, the study of the intercorrelation of the maximum temperature and that of the rainfall, the maximum temperature and the SST shows us that:

- the maximum temperature is 251 days ahead of the rain;
- the maximum temperature is 3 days ahead of the SST.

Finally, we can say that rain, maximum temperature and SST are intercorrelated. The intercorrelation coefficients are respectively:

- Rainfall and maximum temperature: 0.6395;
- SST and Maximum Temperature: 0.8058.

V. Références

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