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A Levenberg-Marquardt Method for Solving the Power Flow Problem in Ill-Conditioned Systems

Ruffin Manasina[1], Roselin Randrianantenaina[2,3], Télesphore Randriamaitso[1], Adolphe Ratiarison[3]

[1] *ISSTM – Higher Institute of Science and Technologies of Mahajanga, Building Ex-LOLO Mahajanga-Be, Madagascar;*
[2] *FSTE – Faculty of Science, Technology and Environment; University of Mahajanga;*
[3] *DyACO – Laboratory of Dynamics of Atmosphere, Climate and Ocean; University of Antananarivo;*

Abstract: *The increase in demand, the growing deployment of electronic and computer equipment, the connection of renewable energy sources and the difficulty of updating existing infrastructures limit the capacities of conventional computing means used in power flow analysis. Recent studies show that new techniques are widely used for solving the power flow problem in well-conditioned and ill-conditioned systems. While the power flow calculation in well-conditioned systems is easily solved, the ill-conditioned power systems can pose difficulties. This paper presents the Levenberg-Marquardt method for solving the power flow problem in ill-conditioned systems. This method is basically the modification of the Newton-Raphson method by adding a Lagrange multiplier in its algorithm. The simulation with the proposed method was performed under Matlab for ill-conditioned IEEE 11-bus, 13-bus, 20-bus and 43-bus test systems. The results obtained are compared with the methods of Newton-Raphson and Runge-Kutta for the number of iterations, the computation time, the tolerance and the convergence error. The results prove that the Levenberg-Marquardt method is efficient and excellent in the case of ill-conditioned power systems.*

Keywords: *Power flow, ill-conditioned systems, Newton-Raphson, Runge-Kutta, Levenberg-Marquardt*

I. Introduction

The power flow (PF) in the power system composed of generators, transmission lines and loads, is probably one of the very important tools for the control, operation and optimization of power systems [1,2]. The main objective of PF is to solve a set of systems of nonlinear equations relating the voltages and powers injected at each bus. Like all nonlinear systems, PF equations can be divided into two categories, namely, well-conditioned and ill-conditioned [3]. Classically, the PF problem in well-conditioned networks is easily solvable using standard solvers such as Newton-Raphson method (NR) [4]. Nevertheless, the PF problem in ill-conditioned systems may pose an additional challenge for conventional techniques [5].

For decades, the PF calculation in ill-conditioned systems was very infrequent. Nowadays, this trend is changing as well as the problem of PF in ill-conditioned systems is more and more frequent in the power system operation [6]. The main reason is the increasing demand for electricity combined with the inflexibility of the existing electricity infrastructure, leading to the low flexibility of the electricity network operator [7]. In addition, the deployment of power electronic equipment accentuates the PF problem in ill-conditioned systems, which contributes to the modification of power line impedances [8]. Therefore, the Jacobian matrix in the conventional NR method becomes an ill-conditioned system. In this context, the development of new efficient and robust approaches for solving the PF problem still remains a major challenge.

This article presents a Levenberg-Marquardt (LM) method, based on the modification of the NR method, for solving the PF problem in ill-conditioned power systems. This method of solving the PF makes it possible to reduce the computation time as well as the number of iterations, on the one hand, to increase the convergence rate, on the other hand, in ill-conditioned power systems. The proposed method is applied on IEEE power systems at 11-bus, 13-bus, 20-bus and 43-bus and the simulation results are compared with NR and RK4 methods.

II. Materials and Methods

2.1 PF Problem and ill-conditioned systems

In a PF problem, the nonlinear equations relating the injected powers and the voltages (magnitudes and angles) at the buses of the power system can be presented in several forms, from the most conventional formulation of residual powers to other alternative approaches, such as those based on current injections [9]. In this article, the formulation and methodologies based on power residuals will be used. The main objective of solving the PF problem is to find the voltages magnitudes and angles at the all buses in power systems. Equations (1) and (2) show the PF equations based on power injections in their hybrid form (polar and rectangular) without line losses [10,11].

$$g(x)=\begin{cases}\Delta P, \text{For all buses}\\ \Delta Q, \text{For PQ buses}\end{cases} \tag{1}$$

$$\left.\begin{aligned}\Delta P_k &= P_k^{sch} - |V_k|\sum_{j=1}^{N}|V_j|\left(G_{kj}\cos\theta_{kj}+B_{kj}\sin\theta_{kj}\right)\\ \Delta Q_k &= Q_k^{sch} - |V_k|\sum_{j=1}^{N}|V_j|\left(G_{kj}\sin\theta_{kj}-B_{kj}\cos\theta_{kj}\right)\end{aligned}\right\} \tag{2}$$

where, g denotes the set of PF equations, x is the PF state vector, $P_k{}^{sch}$ and $Q_k{}^{sch}$ are the active power and reactive injected powers at *k*th bus, respectively, G_{kj} and B_{kj} are the real and imaginary parts of the admittance Y_{kj}, respectively, $V_k=|V_k|\angle\theta_k$ is the complex of the voltage vector at the ***k***th bus, and θ_{kj} is defined as the difference in voltage angle between the ***k***th and ***j***th buses. The terms ΔP_k and ΔQ_k are the difference between the scheduled and calculated values, known as the power residuals. The PF variables to be determined in hybrid coordinates are composed of the voltage angles at PV and PQ buses and the voltage magnitudes at PQ buses, so that the PF state vector is defined by the equation (3).

$$x=\left[\theta_{PV}\ \ \theta_{PQ}\ \ |V_{PQ}|\right]^T \tag{3}$$

where, θ_{PV} is the vector of the voltage angles at PV buses, θ_{PQ} is the vector of the voltage angles at PQ buses, $|V_{PV}|$ is the vector of the voltages magnitudes at PQ buses, and N represent the total number of PV and PQ buses in power systems. The notations of PQ and PV are widely used in PF analysis to characterize the types of buses in power systems. For a brief explanation, the bus in power system can be categorized into three categories (i) PQ buses are load buses in which voltage angle and voltage magnitude are unknown, and active power injections (P) and reactive (Q) are known; (ii) PV buses are generators buses in which voltage angle is unknown, active power injection (P) and voltage angle (θ_{PV}) are known, and reactive power injection is an independent variable; finally (iii) slack bus is a bus in the system whose voltage is fixed, while the power injections are dependent variables.

To simplify the writing of equation (2), the nonlinear equations of PF in hybrid coordinates (polar and rectangular) can be expressed in the following form:

$$g(x)=0 \tag{4}$$

Due to the strong nonlinearities in equation (4), solving the power flow equations is very difficult. Several iterative approaches have been proposed in the literature to solve the PF problem [12,13,14]. The most widely employed in PF analysis is the NR method, which is defined by the following mapping [15]:

$$\left.\begin{aligned} \Delta x^{(h)} &= -\left[g'\left(x^{(h)}\right)\right]^{-1} g\left(x^{(h)}\right) \\ x^{(h+1)} &= x^{(h)} + \Delta x^{(h)} \end{aligned}\right\} \tag{5}$$

where, $g' = \nabla_X g$ is the Jacobian matrix of the PF equation, which is formed by the first-order partial derivatives of the function g with respect to the variables X. The procedure for PF solution by the NR method is continued until the power residuals ΔP_k^h and ΔQ_k^h are less than specified accuracy (Eq.6). Usually, the number of maximum iterations is also limited.

$$\max\left(\text{abs}\left(g(x)\right)\right) < \varepsilon \tag{6}$$

In an ill-conditioned PF problem, the Jacobian matrix is generally singular or quasi-singular and the resolution of the PF equations using the NR solver may fail to converge or have slow convergence rates. Thus, the ill-conditioned power systems is due to the presence of one or more of the following factors, namely, short lines, very high R/X ratio of line, atypical circuit parameter (negative reactance), bad or unfit selection of the slack bus, operation at the limit of stability, radial topology with loops, and choice of voltage initial guess [3]. When the system presented by equation (4) is well-conditioned, the PF solution can be found easily using traditional techniques, especially, NR method. Moreover, the NR method has quadratic and local convergence, which means that the convergence properties are lost if the initial estimate of the state variables X is far away of PF solution. This is the reason why the NR method has convergence difficulties when solving the PF equations in ill-conditioned systems [16].

2.2 Levenberg – Marquardt method

The solution of the PF equations in an ill-conditioned system is very sensitive to the variation of the Jacobian matrix coefficients as well as to the variation of the coefficients of the residual power vector. Most of PF solvers have convergence difficulties. However, the solution of PF in an ill-conditioned systems does exist, but the NR method has difficulties if the flat initial guess of voltage is used [3,17]. For example, at the initial iteration, all voltage magnitudes equal to 1, and all voltage angles equal to 0. Indeed, it is crucial to find the approaches able to solve the problem of PF in ill-conditioned systems.

The Levenberg-Marquardt (LM) method is one of the iterative techniques for solving a PF problem. It consists to minimize the sum of the squares of the residual errors on the active and reactive powers of the buses. Originally, the LM method is a minimization technique widely used in geophysics and electromagnetism [18,19]. For the resolution of the PF, the variables to be determined are the voltage angles at the PV buses, the voltage angles at the PQ buses, and the voltage magnitudes at the PQ buses. In the case of ill-conditioned systems, these variables will be represented by the vector X of equation (3). The constraints are the residual errors on the active power at the PV bus and on the active and reactive power at the PQ bus. Equation (7) represents the constraints on the active and reactive powers mismatches. The column matrix representing the residual errors on the powers at the operating point x_0 is expressed by equation (8).

$$g(x) = [\Delta P \ \Delta Q]^T \qquad (7)$$

$$f_0 = -[g'(x_o)]^{-1} g(x_0) \qquad (8)$$

where $f_0=f(x_0)$ and $g'(x_0)=J_0$ denotes the Jacobian matrix of equation (7) at initial point x_0. At the operating point $x_0+\Delta x$, the residual error on the powers can be approximated by equation (9) using first-order Taylor series expansion. Hence, equation (10) presents the sum of the squares of the residual errors on the active and reactive powers of the buses in power system.

$$f(x_0+\Delta x) = f(x_0)+g'(x_0)\Delta x \qquad (9)$$

$$\begin{aligned} S_1 &= f(x_0+\Delta x)^T f(x_0+\Delta x) \\ &= \Delta x^T g'(x_0)^T g'(x_0)\Delta x + 2\Delta x^T g'(x_0)^T f(x_0) + f(x_0)^T f(x_0) \end{aligned} \qquad (10)$$

The value of the increment Δx of PF variables that minimizes equation (10) cancels the gradient S_1 with respect to the variable x. Equation (11) shows the expression for the null gradient S_1 allowing the value of this increase to be determined. Hence, the solution required to minimize S_1 is given by equation (12).

$$\nabla S_1 = 2g'(x_0)^T f(x_0)\Delta x + 2g'(x_0)^T f(x_0) = 0 \qquad (11)$$

$$\Delta x = -\left[g'(x_0)^T g'(x_0)\right]^{-1}\left(g'(x_0)^T f(x_0)\right) \qquad (12)$$

The result of equation (12) is identical to that obtained by the NR method of equation (5). However, this increment Δx leads to divergence when the initial estimate of is far from the final solution. Moreover, the Jacobian matrix is singular and the cost of the resolution by the method of NR is expensive. To avoid this disadvantage, the LM method offers an alternative by introducing a Lagrange multiplier λ to the sum of the errors of the powers [20]. The introduction of the damping parameter consists in reducing the distance between the final solution and the initial estimate. The new objective function to be minimized can be expressed by equation (13). Hence, the increment required (direction) to minimize S_{LM1} can be obtained by equation (14) [21].

$$S_{LM1} = f(x)^T f(x) + \lambda(x - x_0)^T(x - x_0) \qquad (13)$$

$$\Delta x = -\left[g'(x_0)^T g'(x_0) + \lambda I\right]^{-1}\left(g'(x_0)^T f(x_0)\right) \qquad (14)$$

In LM algorithm, the non-negative of damping parameter is adjusted at each iteration to assure a reduction in the error. There is no exact method for evaluating the damping parameter that will produce optimal convergence for all cases. Indeed, large values of damping factors are preferable when the initial estimate is far from the solution. However, the damping factor can be reduced when the estimate is close to the final solution, so the residual error of the powers becomes smaller. In our case, the LM method program arbitrarily uses equation (15) to determine this factor. Thus, the equation (16) proposes the iterative method of LM allowing to determine the direction Δx at each iteration [21].

$$\lambda_h = \frac{S_h}{1000} = \frac{f(x_h)^T f(x_h)}{1000} \qquad (15)$$

$$\Delta x_h = -\left[g'(x_h)^T g'(x_h) + \lambda_h I\right]^{-1}\left(g'(x_h)^T f(x_h)\right) \qquad (16)$$

Note that, the damping factor in the LM method was introduced to reduce divergence problems and limit the distance between the initial point and the next operation point. If the value of damping parameter is too large, the increment Δx becomes zero and the search for the solution does not progress. When the value is too small or even zero, the solution is identical to that obtained by the NR method and the LM method is potentially very unstable. The PF procedure by the proposed LM method is formally presented in Algorithm 1. The stopping criterion for the algorithm is the convergence tolerance or the maximum number of iterations.

Algorithm 1 : PF solution procedure using LM method	
1:	Set iteration counter: $h \leftarrow 0$
2:	Initial variable guess: $x_h \leftarrow x_0$
3:	**while** $\max\|g(x_h)\| > \varepsilon$ **or** $h \le h_{max}$ **do**
5:	Solve (16)
6:	Update variable: $x_{h+1} \leftarrow x_h + \Delta x_h$
7:	Compute parameter λ_h using (15)
8:	Update iteration counter: $h \leftarrow h+1$
9:	**end while**

The methods of solving the PF problem consist to determine the voltages at the buses in power system in a way to reduce the residual error of the active power and the reactive power to zero. Mathematically, the process of calculation translates into finding the roots of a system of nonlinear equations. Alternatively, the search for the solution of the EP can be carried out by determining the voltages so as to minimize the sum of the squares of the residuals on the active and reactive powers.

The PF methods proposed in this article take into account the limits of reactive power generators and equipment controls. A technique commonly used in these methods is to check at each iteration the reactive power produced on the PV buses, and to switch the PV to a PQ bus if the reactive power limit has been exceeded.

III. Results and discussion

In this section, the performances of the LM method in terms of number of iterations, computation time, and convergence tolerance are compared with those of the NR and RK4 methods, which are widely employed to solve the EP equations. In order to check the efficiency and performance of the proposed LM method, the ill-conditioned systems of IEEE 11-bus, IEEE 13-bus, IEEE 20-bus, and IEEE 43-bus were considered. Topologies and parameters of ill-conditioned IEEE 11-bus, IEEE 13-bus, and IEEE 43-bus networks can be found in [22]. Additionally, details of the IEEE 20-bus ill-conditioned systems are available in [23]. All the simulations of the ill-conditioned systems have been implemented using Matlab R2018a by a Lenovo PC with an Intel® CoreTM 2 Duo CPU P8600 2.4 GHz processor and 4 GB of RAM, under Ubuntu 18.94 LTS.

Table 1 provides the computation time (in milliseconds) and total number of iterations for the four ill-conditioned systems IEEE 11-bus, IEEE 13-bus, IEEE 20-bus, and IEEE 43-bus with the different PF solvers. These computations times were obtained as the average over 200 runs of the PF programs, in order to avoid the influence of other computer activities. The convergence tolerance was set to 10^{-7}. According to this table, the LM method improves the efficiency and robustness of the NR method. For example, the use of the LM method for solving the PF equations in the IEEE 43-bus power system reduces the computation time by 40% and 94.2% compared to the methods of NR and RK4, respectively. It should be noted that the GS method failed the convergence in all studied cases, and the NR method have a difficulty to reach convergence in the 11-bus network. When the size of the ill-conditioned system is large, the difference between the computation times of the different methods is slightly small. For the IEEE 13-bus system, the computation time of the LM method is about 32% of the NR method. The LM method requires less number of iterations to reach convergence. For example, the use of the LM method in an ill-conditioned 43-bus system reduces the computation time by 37.5% and 77.3% compared with the NR and RK4 methods, respectively. We observed that the number of iterations required by the proposed LM method is almost independent of the network size. This number is smaller compared to the RK4 method, but it is slightly low compared to that of the NR method. However, the convergence of these methods depends on the initial values of the voltages. In our case, we have used the initial values of the default voltages provided by the ill-conditioned IEEE systems [22,23].

Table 1 : Computation time (milliseconds) and number of iterations for the different PF methods

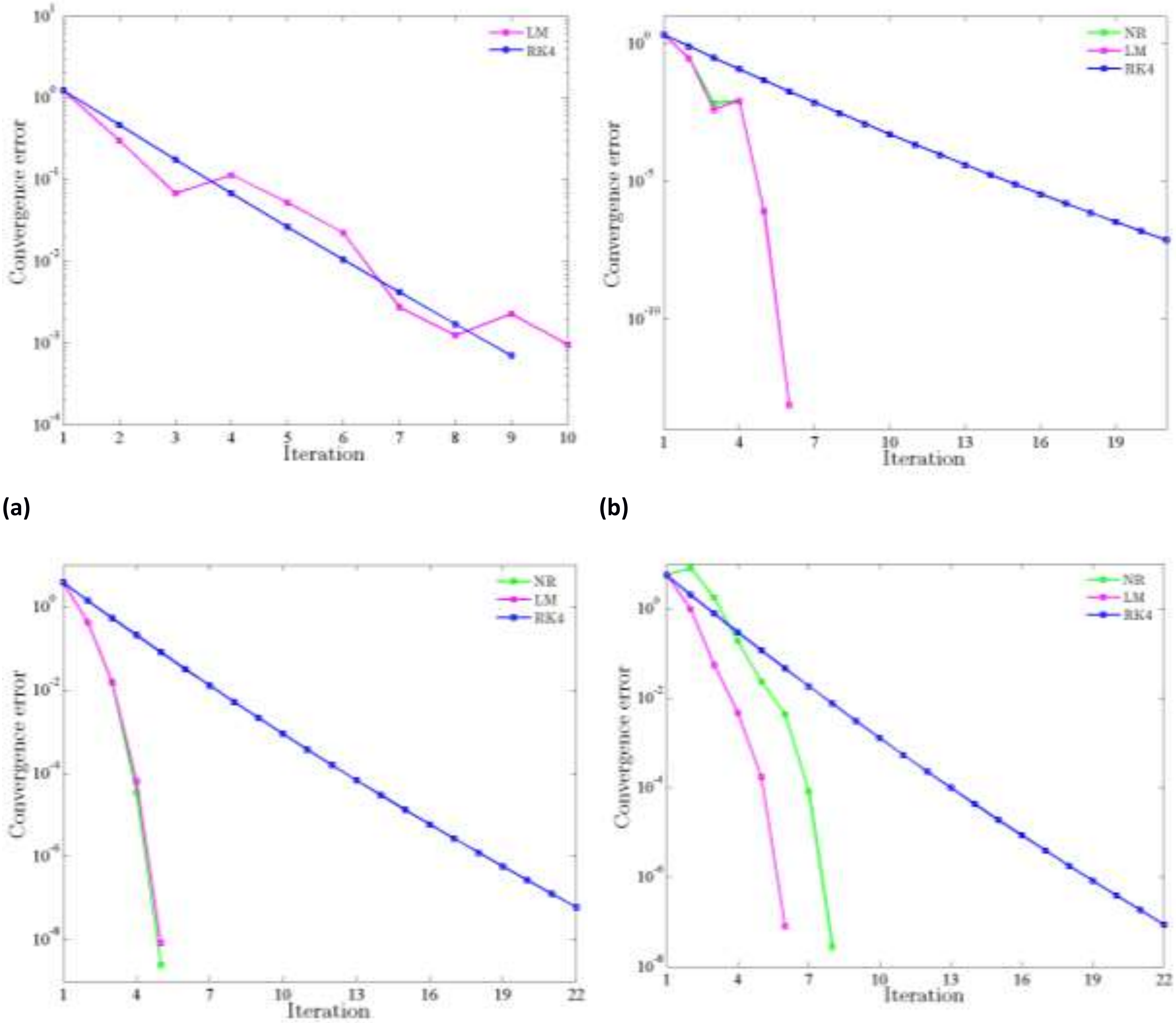
Test system	GS method [24]		NR method [24]		RK4 method [24]		LM method	
	CPU time	Iter. #	CPU time	Iter.#	CPU time	Iter.#	CPU time	Iter.#
IEEE 11-bus	Fail	Fail	Fail	Fail	92,31	9	41,18	10
IEEE 13-bus	Fail	Fail	27,78	5	206,92	21	18,88	5
IEEE 20-bus	Fail	Fail	37,42	6	581,62	22	35,87	6
IEEE 43-bus	Fail	Fail	324,48	8	3332,18	22	194,06	5

In order to better measure the performance of the LM method, the results of the simulations in Table 2 are then obtained by varying the precision criteria from 10^{-1} to 10^{-8}. For the case of the 11-bus system, the transmission lines parameters are given in the form of admittance with only 3 decimal places, and we were able to make the system converge only with a precision of 10^{-3}. From the convergence tolerance value $\varepsilon = 10^{-3}$, the number of iterations of the NR method is nearly constant. In all the cases of the studied systems, the variation of the precision does not have a major influence on the number of iterations of the LM method. So this method has the best performance in ill-conditioned systems compared to other methods.

Table 2. C Comparison number of iterations and computational accuracy using the NR, LM and RK4 methods

Tolerance	**11-bus**			**13-bus**			**20-bus**			**43-bus**		
	NR	**RK 4**	**LM**	**NR**	**RK 4**	**LM**	**NR**	**RK4**	**LM**	**NR**	**RK 4**	**LM**
0.1	3	4	3	3	5	3	3	5	3	5	6	3
0.01	7	7	7	3	7	3	4	8	4	6	8	4
0.001	9	9	10	5	10	5	4	10	4	7	11	5
0.0001	-	9	10	5	12	5	4	13	4	7	13	6
0.00001	-	9	10	5	15	5	5	16	4	8	16	6
0.000001	-	9	10	5	18	5	5	19	5	8	19	6
0.0000001	-	9	10	6	21	6	5	22	5	8	22	6
0.00000001	-	9	10	6	24	6	5	25	5	9	25	7

Figure 1 shows the comparison of convergence errors as a function of number of iterations with the three methods for tolerance $\varepsilon = 10^{-7}$. From this figure, the LM method offers the possibility of achieving very fast convergence compared to other methods. In a 43-bus power system, the NR, RK4, and LM methods converged at 8, 22, and 6, respectively. The LM method reaches the convergence of the system from the ***6***th iteration of the calculation program. This rapid convergence is obtained because of the introduction of the damping parameter λ in the LM method. However, the NR method may present convergence difficulties in the 11-bus system. Moreover, the topology of this transmission system is radial and has only one generator. At the end of each system convergence process, the residual powers of the different methods are presented in Table 3. Indeed, the convergence error of the LM method is significantly low in all cases of the power systems studied.



(a) (b) (c) (d)

Figure 1. Comparison of convergence profiles for the power systems using the three methods with $\varepsilon = 10^{-7}$: (**a**) IEEE 11-bus; (**b**) IEEE 13-bus; (**c**) IEEE 20-bus; (**d**) IEEE 43-bus.

Table 3. Convergence error for complete process using the different PF solvers

Method	11-bus	13-bus	20-bus	43-bus
NR	-	7.4746×10^{-14}	2.4513×10^{-9}	2.7527×10^{-8}
RK4	7.0054×10^{-4}	7.4379×10^{-8}	6.0038×10^{-8}	8.7214×10^{-8}
LM	9.5325×10^{-4}	$7,4849\times10^{-14}$	8.4326×10^{-9}	$7,8639\times10^{-8}$

The voltage profiles for ill-conditioned IEEE 11-bus, 13-bus, 20-bus, and 43-bus systems using the NR, RK4, and proposed LM methods are presented in Figure 2. The result of the LM method makes it possible to reduce the difference between the calculated voltages and their initial values necessary to start the algorithm. For example, in a 43-bus system, the NR, RK4, and LM methods calculated the voltage at bus 43 as 1.0763, 1.0763, and 1.0404 per unit, respectively. While the initial voltage value at bus 43 has been adjusted to 1 per unit.

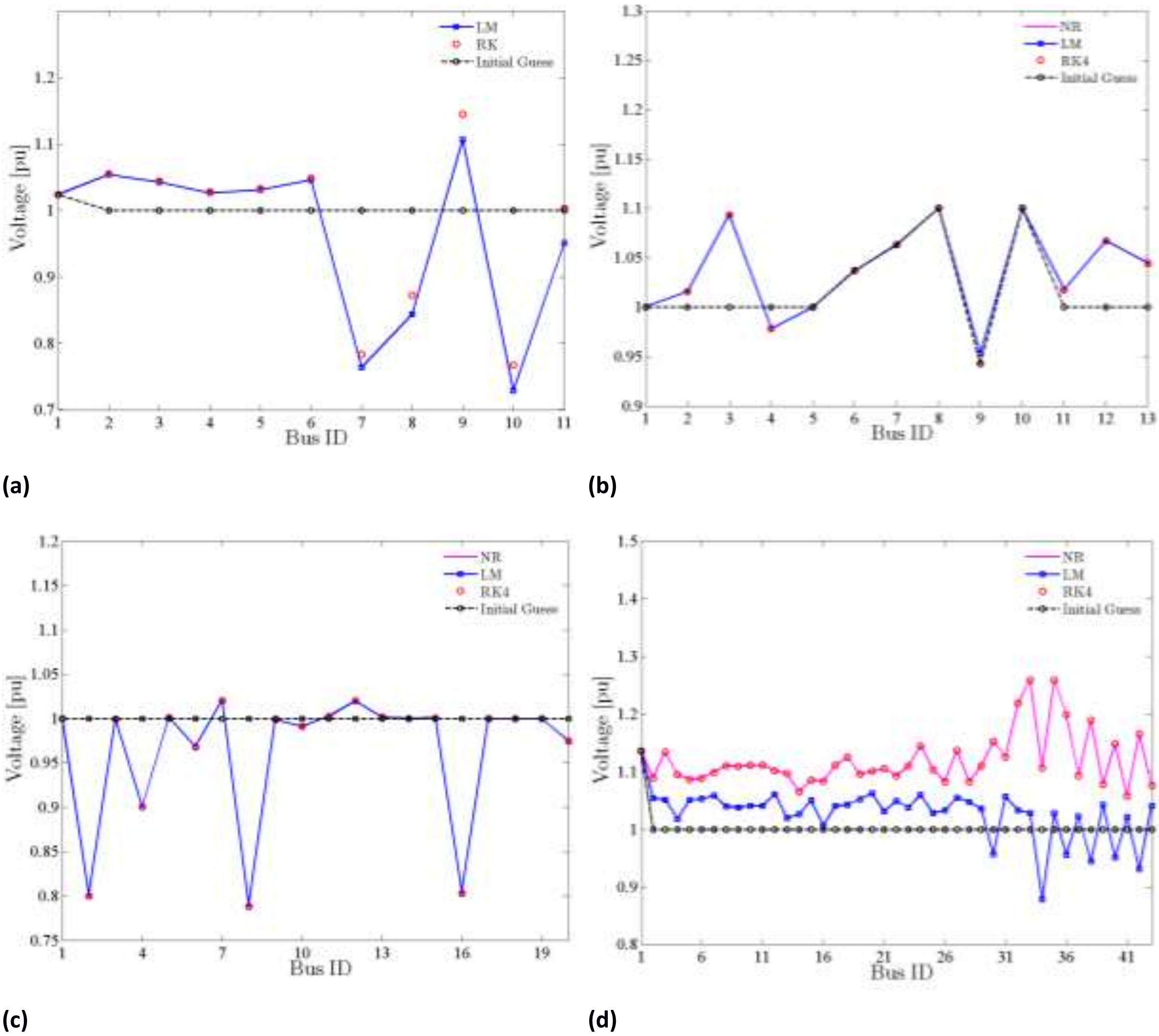


Figure 2. Comparison of voltage profiles for the power systems with convergence tolerance $\varepsilon = 10^{-7}$: (**a**) IEEE 11-bus; (**b**) IEEE 13-bus; (**c**) IEEE 20-bus; (**d**) IEEE 43-bus

The characteristics of the curves obtained for the three methods have the same tendencies. The results of PF calculation in ill-conditioned systems show that the estimate of the initial guess of the voltages is close to the final solutions found by the methods of RK4 and LM. The proposed LM method always gives the best results compared to other methods.

IV. Conclusion

In this paper, the LM method has been proposed to solve the PF problem in ill-conditioned power systems. This method is the improvement of the NR method by adding a damping parameter in the formulation of the PF problem. The formulation and algorithms associated with PF problem have been presented and applied to ill-conditioned IEEE 11-bus, 13-bus, 20-bus, and 43-bus test power systems. The simulation results clearly show that the proposed LM method is more competitive than the other traditional PF solvers in the ill-conditioned power systems. The computation time and the number of iterations in this proposed method are reduced compared with the NR and RK4 methods. Moreover, the number of iterations required for the convergence of the system does not vary much with the increase of the computational precision. The performance of the LM method is better than the performance of the NR and RK4 methods.

The LM method offers another approach to solving PF equations in ill-conditioned systems with a simplified design, fast convergence, low number of iterations, and less computation time. Future works should be focused on application of this method in large dimensional ill-conditioned power systems.

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