American Journal of Sciences and Engineering Research

E-ISSN -2348 – 703X, Volume 5, Issue 3, 2022

Power Flow Calculation for Well-Conditioned Systems Using the 4th Order Runge - Kutta Method

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Abstract: *Power flow is an essential computational tool in planning and operation of power systems. It is used to determine the steady-state electrical state of the network so as to reduce the residual error of the active and reactive power to zero. This paper presents a method for solving the power flow problem by the fourth order Runge - Kutta method. This method makes it possible to calculate the voltages at the different buses, the electric currents, the active and reactive powers supplied by each source and the transits of the active and reactive powers in the lines. The fourth order Runge - Kutta method is obtained by making the analogy between the Newton - Raphson method and the explicit Euler numerical integration method. The simulations were performed using Matlab for the test cases of IEEE 9-bus, 14-bus, 30-bus, 57-bus, and 118-bus systems. The results obtained were compared with the Gauss-Seidel and Newton-Raphson methods for the number of iterations, computational time, tolerance value, and convergence error. The results of analyses show that the Runge - Kutta method is efficient and excellent in the case of large-scale well-conditioned systems.*

Keywords: *Gauss – Seidel, Newton – Raphson, Runge – Kutta, Power-flow analysis, well– conditioned systems*

I. Introduction

In an electrical transmission system, energy flows from the generator to the load via different lines in the power system. Power Flow (PF) calculation is an important tool for the manager and operator of the electric power system. It provides a view of the network infrastructure in order to reinforce, modify and adapt the network according to consumption [1]. The objective of PF analysis is to determine the complete electrical state of the network in steady state, i.e. the voltages in all the buses, the transits of active and reactive powers in all the lines, the line losses, etc. from the consumptions and productions specified in these buses.

PF equations are non-linear and must be solved by iterative techniques using numerical methods. Solving these equations has led many researchers to find simpler and faster numerical methods in order to improve their convergence rate, reduce the computing time and save some computer memory. During the last three decades, many methods have been used to solve the PF problem [2]. The most commonly used iterative methods are the Gauss-Seidel (GS) and Newton-Raphson (NR) methods [3,4]. However, with the industrial development in the society involving the increase of the large power systems to be processed, some methods fail to converge to a correct solution.

In this paper, we focus on the Runge-Kutta method of order 4 (RK4). This method of solving the power flow problem is based on the analogy between the NR method and the ordinary differential equations.

 Received-13-06-2022, Accepted- 21-06-2022

Simulation results of the RK4 method are compared with the traditional methods GS and NR for the IEEE 9-bus, 14-bus, 30-bus, 57-bus and 118-bus systems.

II. Materials and Method

2.1 Bus classification

A bus in power system is a busbar to which one or more transmissions lines, loads and generators are connected. In power system, the buses are associated four quantities like voltage magnitude $|V|$, the phase angle of voltage θ, active power P and reactive power Q [5,6]. In load flow, only two of these four quantities are known at a bus, the other two must be determined during the calculation [7,8]. The classification of power system buses into three categories based on the variables used as shown in Table 1.

Table 1. Classification of power system buses

Bus code	Bus Type	Notation	Know variables	Unknow variables
1	Reference bus	Slack bus	$\|V\|, \theta$	P, Q
2	Generator bus	PV bus	$P, \|V\|$	Q, θ
3	Load bus	PQ bus	P, Q	$\|V\|, \theta$

2.2 PF problem and well-conditioned cases

The PF analysis is used to determine the complex voltages of the power system at different bus, the powers transited from one bus to another, the injected powers at a bus, and the active and reactive losses power [9]. The first step in power flow calculation is to formulate the admittance matrix of the power system using data from transmission lines, capacitors, and transformers. The solution of the steady state power flow problem is based on the following linear equation system:

$$I = YV \tag{1}$$

where I is the complex vector of the bus currents injection; Y is the matrix of complex admittances and V is the complex vector of nodal voltages.

The net complex apparent power and current injection into any bus (k) can be expressed, respectively, by the following equation:

$$\left.\begin{aligned} S_k &= V_k I_k^* \\ I_k &= \sum_{j=1}^{N} \left(G_{kj} + jB_{kj}\right) |V_k| \angle \theta_k \end{aligned}\right\} \tag{2}$$

where, S_k is the complex net injected power into the *k*th bus, V_k is the complex voltage at *k*th bus, I_k is the current injected at *k*th bus, I_k^* is the conjugate of I_k, N is the total number of bus bars, G_{kj} and B_{kj} the real and imaginary parts of the admittance elements Y_{kj}, $|V_k|$ is the voltage magnitude at *k*th bus, and θ_k is the voltage angle at *k*th bus.

This equation shows that the active and reactive powers injected at the bus are a function of the magnitude and angle of the voltages at the other bus. Equations (3) and (4) represent the PF equations at *k*th bus of a power system [10].

$$P_k = P_{Gk} - P_{Dk} = |V_k| \sum_{j=1}^{N} |V_j| \left(G_{kj} \cos\theta_{kj} + B_{kj} \sin\theta_{kj}\right) \tag{3}$$

$$Q_k = Q_{Gk} - Q_{Dk} = |V_k| \sum_{j=1}^{N} |V_j| \left(G_{kj} \sin\theta_{kj} - B_{kj} \cos\theta_{kj}\right) \tag{4}$$

where, P_k and Q_k are the net active and reactive powers injected into the bus k, P_{Gk} and Q_{Gk} are the active and reactive powers supplied by the generators to the bus k, P_{Dk} and Q_{Dk} are the active and reactive powers consumed by the loads connected to the bus k, and θ_{kj} is the angle between the complex voltages V_k and V_j.

When the PF equations are well-conditioned, the PF solution exists and is reachable using a flat initial guess. For example, all load voltage magnitudes equal to 1 and all bus voltage angles equal to 0 in bus system. Indeed, the PF solution can be easily found by using conventional solvers, such as, Gauss-Seidel and Newton-Raphson methods [11]. The well-conditioned cases are still the most common situation.

Regarding the PF analysis, the analytical resolution of the nonlinear equations that relate the nodal voltages with the power injections is very difficult. For this reason, many iterative numerical methods are suitable in the literature to solve the PF problem [12,13]. In this paper, three such as methods are studied for solving the PF problem: the Gauss-Seidel, Newton-Raphson and Runge - Kutta.

2.3 Gauss-Seidel method

The Gauss-Seidel (GS) method is one of the simplest iterative methods used for solving nonlinear power flow equations [14,15]. This method consists of successively calculating the voltage at each bus of the network. For the PQ bus, the real and reactive powers are known. Thus, if the initial bus voltage is given, we can use the equation (5) to perform the iteration calculation [16].

$$V_k^{(h+1)} = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{\left(V_k^{(h)}\right)^*} - \sum_{j=1}^{k-1} Y_{kk} V_j^{(h+1)} - \sum_{j=k+1}^{N} Y_{kj} V_j^{(h)}\right] \tag{5}$$

For the PV bus, only active power injection and magnitude voltage are specified and the reactive power injection is unknown. Therefore, the injected reactive power is calculated by the following equation:

$$Q_k^{(h+1)} = -imag\left(\left(V_k^{(h)}\right)^*\left(Y_{kk}V_k^{(h)} + \sum_{j=1}^{k-1} Y_{kj}V_j^{(h+1)} + \sum_{j=k+1}^{N} Y_{kj}V_j^{(h)}\right)\right) \tag{6}$$

During the iteration process, if the calculated value of reactive power does not violate any of the specified limits, then the voltage angle can be determined by equation (7). However, if the calculated value of reactive power violates one of the specified limits, then the corresponding PV bus is treated as the PQ bus. The convergence of the system is reached when the condition specified in equation (8) is satisfied.

$$V_k^{(h+1)} = \left|V_{k,Scheduled}\right| \frac{V_i^{(h+1)}}{\left|V_i^{(h+1)}\right|} \tag{7}$$

$$\max\left|V^{(h+1)} - V^{(h)}\right| \le \varepsilon \tag{8}$$

2.4 Newton- Raphson method

The PF problem can also be solved by the Newton-Raphson method. This method allows to strongly reducing the computational time of the PF problem, especially for large power system. However, it requires more mathematical calculations and consequently more computational time per iteration than the Gauss-Seidel method, while it converges more quickly even for large-scale well-conditioned system [17]. The active and reactive injected powers at each bus of the power system can be expressed by equation (9).

$$\begin{aligned} P_k &= |V|_k \sum_{j=1}^{N} |V_j| \left(G_{kj}\cos\theta_{kj} + B_{kj}\sin\theta_{kj}\right) \\ Q_k &= |V|_k \sum_{j=1}^{N} |V_j| \left(G_{kj}\sin\theta_{kj} - B_{kj}\cos\theta_{kj}\right) \end{aligned} \tag{9}$$

These power equations are nonlinear and can be solved using the Taylor series and using the first-order series approximation [18]. The Jacobian matrix gives the relationship between small changes in magnitude $\Delta|V|$ and angle $\Delta\theta$ of voltage and small changes in active ΔP and reactive power ΔQ .

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}=-\begin{bmatrix}\left(\frac{\partial P}{\partial\theta}\right) & \left(\frac{\partial P}{\partial|V|}\right)\\ \left(\frac{\partial Q}{\partial\theta}\right) & \left(\frac{\partial Q}{\partial|V|}\right)\end{bmatrix}\begin{bmatrix}\Delta\theta\\ \Delta|V|\end{bmatrix} \tag{10}$$

In equation (10), the Jacobian matrix is also denoted as follows:

$$[J]=\begin{bmatrix}\left(\frac{\partial P}{\partial\theta}\right) & \left(\frac{\partial P}{\partial|V|}\right)\\ \left(\frac{\partial Q}{\partial\theta}\right) & \left(\frac{\partial Q}{\partial|V|}\right)\end{bmatrix} \tag{11}$$

The Jacobian matrix J contains the partial derivatives of the active and reactive powers with respect to the angles and the magnitudes of the tensions. The elements of this matrix are the partial derivatives of the active and reactive powers, evaluated at $\Delta\theta_k^{(h)}$ and $\Delta|V_k|^{(h)}$. Solving the linear system of equation (10) provides the vectors $\Delta\theta$ and $\Delta|V|$, which can be written in the following reduced form:

$$\begin{bmatrix}\Delta\theta\\ \Delta|V|\end{bmatrix}=-[J]^{-1}\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} \tag{12}$$

The terms $\Delta P_k^{\,h}$ and $\Delta Q_k^{\,h}$ are the difference between the scheduled net powers and those of calculated at the iteration h, also called the active and reactive powers mismatch, given by equation (13).

$$\begin{cases}\Delta P_k^{(h+1)}=P_k^{(h)}-P_k^{sch}\\ \Delta Q_k^{(h+1)}=Q_k^{(h)}-Q_k^{sch}\end{cases} \tag{13}$$

Each iteration, the voltages magnitudes and angles at all buses are estimated by the equation (14). The calculation procedure is repeated until the residual errors for the powers and for all the busbars are within the specified tolerances.

$$\begin{cases}\theta^{(h+1)}=\theta^{(h)}+\Delta\theta^{(h)}\\ |V|^{(h+1)}=|V|^{(h)}+\Delta|V|^{(h)}\end{cases} \tag{14}$$

2.5 Runge – Kutta method

The Runge-Kutta method is a technique of one-step numerical schemes for numerically solving first Order Differential Equations (ODE). Originally, Euler's method is the simplest method for numerically solving differential equations. However, the explicit Euler's method can be numerically unstable, even at variable time steps. Given the analogy between power flow equations and ODE, all numerical integration methods can be applied to solve PF equations [19]. But, it is necessary to evaluate an efficient integration method to solve them. The analogy between the Newton-Raphson method and the forward Euler method used for numerical integration is straightforward if function f is defined as follows:

$$f(X)=\frac{\Delta X}{\Delta t}=-[J_0]^{-1}F_0 \tag{15}$$

The estimate of the integral of the function for f a time step of 1 second $(\Delta t=1)$ can be expressed by the following equation:

$$\Delta X^{(h)} = f\left(X^{(h)}\right)\Delta t$$
$$X^{(h+1)} = X^{(h)} + \Delta X^{(h)} \quad (16)$$

This approach can be extended using the Runge-Kutta numerical integration method of order 4 (RK4)[20]. The explicit relationship of the RK4 method is as follows:

$$\begin{aligned} k_1 &= f\left(X^{(h)}\right) \\ k_2 &= f\left(X^{(h)} + 0.5\Delta t\, k_1\right) \\ k_3 &= f\left(X^{(h)} + 0.5\Delta t\, k_2\right) \\ k_4 &= f\left(X^{(h)} + \Delta t\, k_3\right) \\ X^{(h+1)} &= X^{(h)} + \Delta t\left(k_1 + 2k_2 + 2k_3 + k_4\right)/6 \end{aligned} \quad (17)$$

where Δt is a given the time step. The time step can be adjusted according to the estimated truncation error of the integration method. Discussions on estimating the truncation error of RK4 are offered in [21]. For the RK4 method, the adopted the truncation error is based on the half-step method, as follows:

$$\xi = \max\left(\left|k_2 - X\right|\right) \quad (18)$$

In this approach, the equation (19) show the time step can be adjusted based on the following simple heuristic rules [20,21].

$$\begin{aligned} &\textit{if } \xi > 0.01 \textit{ then } \Delta t \leftarrow \max\left\{0.985\ \Delta t,\ 0.75\right\} \\ &\textit{if } \xi \le 0.01 \textit{ then } \Delta t \leftarrow \min\left\{1.015\ \Delta t,\ 0.75\right\} \end{aligned} \quad (19)$$

The time step Δt is variable according to these rules. This time step is increased if the truncation error is greater than a given threshold and it is decreased if the truncation error is less than a given threshold. The minimum value of the time step is limited to 0.75. Experiments have been made showing that all the different families of Runge-Kutta methods can be used to solve the PF calculation, and these methods are more stable than the explicit Euler method [20]. But in our study, we proposed to use the Runge-Kutta method of higher order, that of the 4th order. The pseudocode for PF calculation using RK4 method is summarized in Algorithm 1. Regarding this iterative procedure, the computing Jacobian matrix $J\left(X\right)$ needs to update 4 times per iteration, and also the calculation of the inverse of this matrix requires additional computation time [22]. In the case of algorithm, the voltage angles at PV and PQ buses, along voltage magnitudes at PQ buses constitute the PF variables.

Algorithm 1 PF solution procedure using RK4 method

1: Set iteration counter: $h \leftarrow 0$
2: Initial variable guess: $X^{(h)} \leftarrow X^{(0)}$
3: Set time step $\Delta t \leftarrow 1$
4: **while** $\max\left\{\text{abs}\,(\Delta X^{(h)})\right\} > \varepsilon$ **or** $h \ge h^{\max}$ **do**
5: Solve (17)
6: Update time step Δt using (19)
7: Update iteration counter: $h \leftarrow h+1$
8: **end while**

The three numerical methods discussed in this article directly take into account the limits of reactive power generators and equipment controls. During the iteration process, a technique commonly used in these methods consists to check the value of the reactive power produced on the PV buses, and switching the PV bus to a PQ bus if the reactive power limit has been exceeded.

III. Results and Discussion

3.1 Materials and systems details

All simulations of IEEE 9-bus, IEEE 14-bus, IEEE 30-bus, IEEE 57-bus, and IEEE 118-bus systems were performed on a LENOVO personal computer, Intel® Core™ 2 Duo CPU P8600 2.4 GHz and 2 GB of RAM, equipped with Ubuntu 18.94 LTS operating system. The PF programming codes were developed in a Matlab R2018a environment. The topologies, transmission lines data, and bus data of these systems test from the Institute of Electrical and Electronics Engineers (IEEE) can be found in [23,24].

In order to evaluate the efficiency and performance of the proposed method, the results of the RK4 method are compared with those obtained by the two classical GS and NR numerical methods, which are commonly used to solve the PF equations in well-conditioned case systems.

3.2 Computation time

Along with the iteration number, the computation time is widely used as indicator for comparing the performance of PF solvers. The computation time corresponds to the time spent in execution of PF solver that we used for the test. In all cases studies, we executed the algorithms in 100 times and the average value was calculated. The average computation time for the IEEE 9-bus, 14-bus, 30-bus, 57-bus, and 118-bus systems with the different solving methods are given in Table 2. Indeed, the convergence tolerance (accuracy) is set to 10^{-7}. As observed, using the RK4 method consumes more computation time when comparing with the others methods. For example, the use of RK4 for solving the PF equations of IEEE 30-bus test system, gives the computation time of 10 and 18 times higher than those of GS and NR methods, respectively. Our finding shows that the smaller the size of power system, the smaller the computational time difference will be. For example, for IEEE 118-bus test system, the computation time of RK4 method is estimated to be 1.13 times higher than GS. The RK4 method requires a lot of additional computation time to achieve convergence. In contrast, there is no risk of divergence for large-scale well-conditioned system.

Table 2: Computation time (seconds) for the different PF solutions

Method	9-bus	14-bus	30-bus	57-bus	118-bus
GS	0.015	0.025	0.159	0.719	15.734
NR	0.014	0.021	0.085	0.687	2.217
RK4	0.119	0.252	1.541	5.898	17.830

3.3 Number of iterations

In all the IEEE systems studied, the accuracy of the calculation is set to 10^{-7}. Table 3 reports the number of iterations of the three cited methods to solve the PF equations for 9-bus, 14-bus, 30-bus, 57-bus, and 118-bus IEEE systems. We observed that the number of iterations required for the proposed RK4 method is almost independent of the power system size. This number is higher compared to NR method, but it is slightly low compared to that of the GS method. The number of iterations of the NR method increases slightly with the power system size. The GS method, on the other hand, is less robust and generally converges more difficult when the size of system is high. The NR method requires less number of iterations to reach convergence. They require about 12 iterations for convergence in large systems. However, the convergence of these two methods depends on the initial guess of the voltages. In our case, we used the initial values of the default voltages provided by the IEEE bus systems.

Table 3. Number of iteration for the different PF solution

Method	**9-bus**	**14-bus**	**30-bus**	**57-bus**	**118-bus**
GS	98	129	360	756	4700
NR	6	7	6	12	12
RK4	22	21	23	23	23

3.4 Convergence tolerance value

The convergence tolerance error value is an important factor that determines the accuracy of the PF solution. In order to better measure the performance of the RK4 method, simulations are then performed with different convergence tolerance values. The tolerance value is varied from 10^{-1} to 10^{-8}. The results of the simulations are shown in Table 4. For the GS method, while the computational accuracy is high, the number of iterations is large. This finding is very noticeable in the case where the size of system is large. The number of iterations of the NR method is almost constant with the variation of the computational accuracy. From the convergence tolerance value of 0.001, the number of iterations of the RK4 method is almost constant.

Table 4. Comparison number of iterations and computational accuracy using the GS, NR and RK4 methods

Tolerance	9-bus			14-bus			30-bus			57-bus			118-bus		
	GS	NR	RK4	GS	NR	RK4	GS	NR	RK4	GS	NR	RK4	GS	NR	RK4
0.1	2	4	5	2	4	5	2	4	6	2	4	6	2	4	7
0.01	3	4	8	4	4	7	4	4	9	14	10	9	6	10	9
0.001	7	5	10	25	4	10	8	5	11	139	11	11	29	11	12
0.0001	10	5	13	51	7	12	117	5	14	237	11	14	59	11	15
0.00001	40	5	16	77	5	15	199	5	17	308	11	17	1480	11	17
0.000001	69	5	19	103	7	18	280	5	20	590	11	20	3119	12	20
0.0000001	98	6	22	129	7	21	360	12	23	756	12	23	4700	12	23
0.00000001	127	6	27	157	7	24	439	6	24	908	12	26	6276	12	27

3.5 Convergence errors

The number of iterations is determined by the convergence characteristic of the method. The convergence features of PF solutions are described by the maximum mismatch at each calculation step as a function of number of iterations. The tolerance value chosen for these methods is 10^{-7}. At the end of each iteration process, the maximum convergence errors with the five IEEE bus systems are presented in Table 5. Based on that, the RK4 method gives the smaller errors compared to the GS method. Moreover, the convergence error of NR method is significantly low in all IEEE bus systems studies.

Table 5. Convergence error for complete the process using the different PF solutions

Method	9-bus	14-bus	30-bus	57-bus	118-bus
GS	9.6486×10^{-8}	9.8530×10^{-8}	9.8718×10^{-8}	9.8718×10^{-8}	9.9935×10^{-8}
NR	1.6875×10^{-14}	5.1878×10^{-9}	7.4152×10^{-14}	2.1316×10^{-14}	2.0833×10^{-12}
RK4	5.5311×10^{-8}	6.5760×10^{-8}	5.7502×10^{-8}	5.0269×10^{-8}	9.9928×10^{-8}

Figure 1 shows the comparison of the convergence errors with the studied methods. In this figure, the convergence error drops rapidly for all test systems performed from the 1st iteration, and the NR method converges very fast. In all cases, the proposed RK4 method offers the possibility to reach the convergence quite fast even though it performed 4 times the Jacobian matrix inversion per iteration. The GS method may have convergence difficulties on the one hand, and on the other hand, it may converge to physically infeasible solutions.

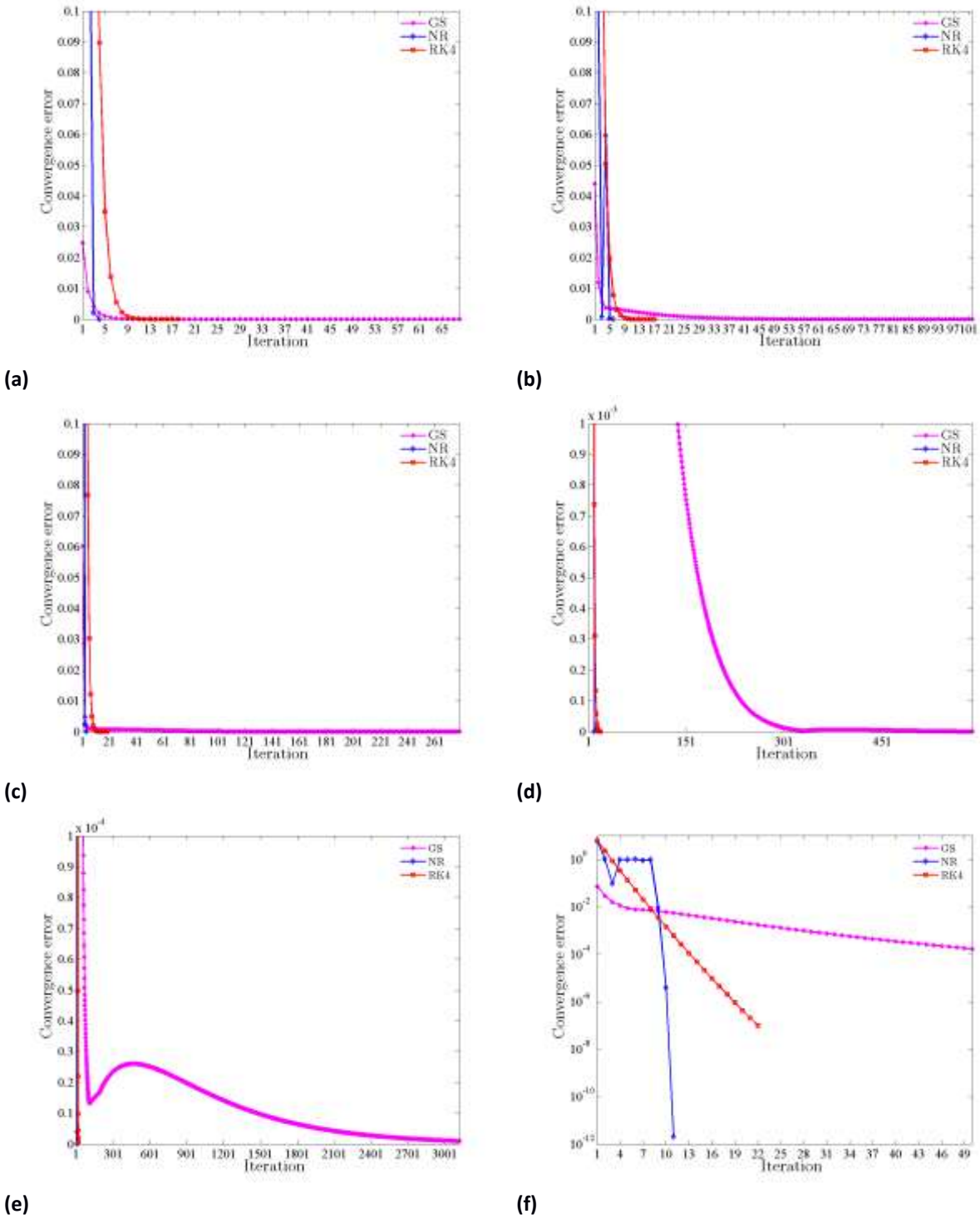

Figure 1. Comparison of convergence profiles for the power systems using the three methods with $\varepsilon = 10^{-7}$: (**a**) IEEE 9-bus; (**b**) IEEE 14-bus; (**c**) IEEE 30-bus; (**d**) IEEE 57-bus ; (**e**) IEEE 118-bus ; (**f**) IEEE 118-bus (zoom).

3.6 Voltage profiles

The voltage profiles for IEEE standard 9-bus, 14-bus, 30-bus, 57-bus, and 118-bus test systems versus bus numbers using the GS, NR, and proposed RK4 methods are shown in Figure 2 and Figure 3. It is clearly seen that, the result of the RK4 method makes it possible to reduce the difference between the calculated voltages and their initial values. For example, in a 57-bus network, the GS, NR, and proposed RK4 methods calculated the voltage at bus 57 as 1.4917, 1.2182, and 1.1727 pu, respectively. The initial voltage value at bus 57 was set

to 1 pu. The RK4 method successfully solved the IEEE 57-bus case (Figure 2.**d**), and it converged to the low voltage solution while GS method converged to the high voltage solution.

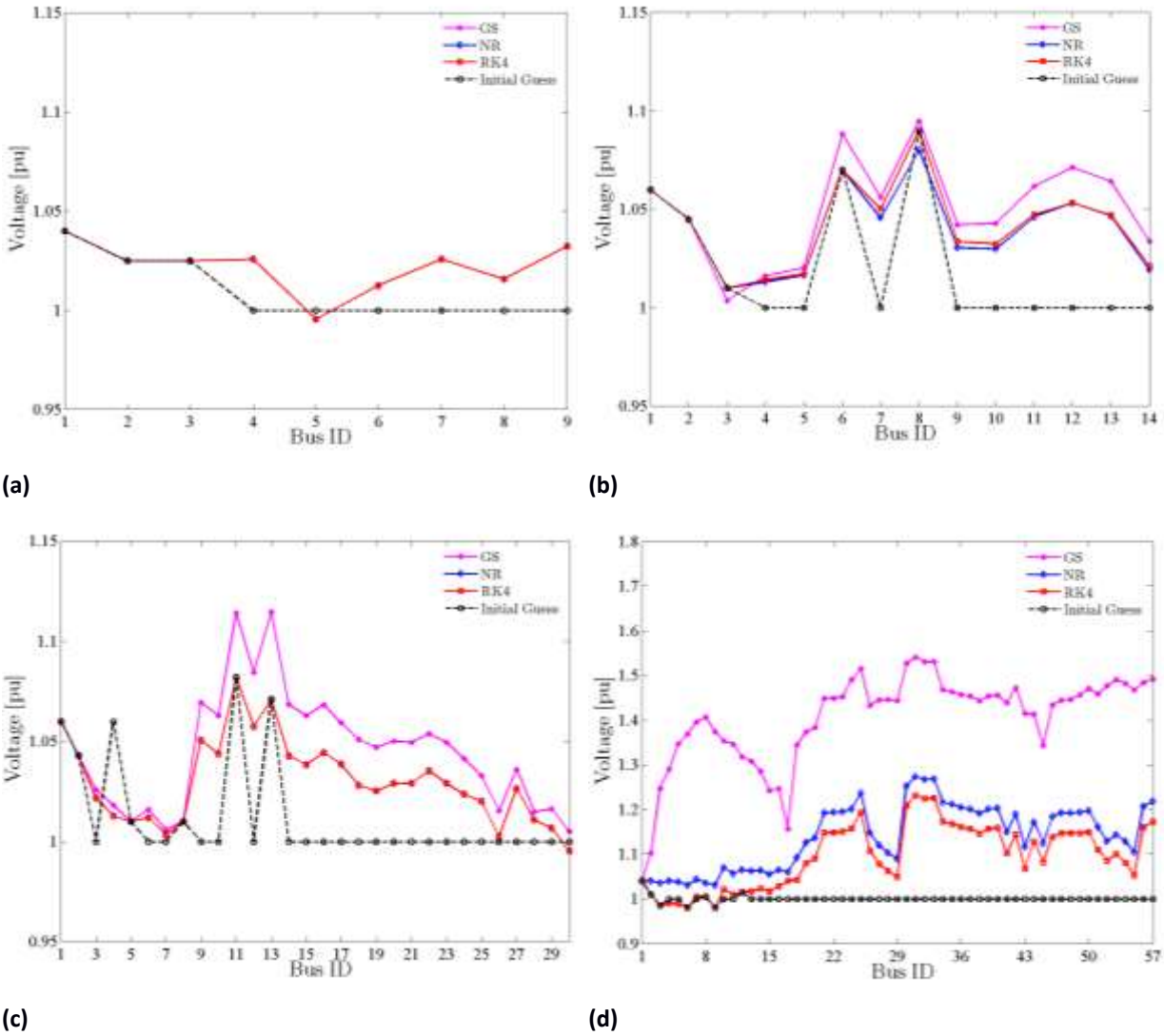

Figure 2. Comparison of voltage profiles for the power systems with convergence tolerance $\varepsilon = 10^{-7}$: (**a**) IEEE 9-bus; (**b**) IEEE 14-bus; (**c**) IEEE 30-bus; (**d**) IEEE 57-bus.

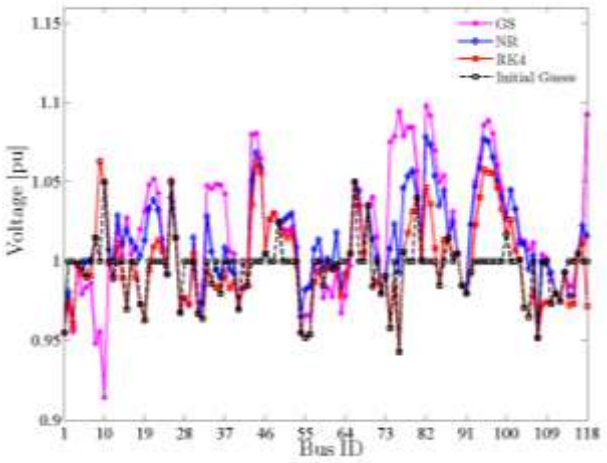

Figure 3. Comparison of voltage profiles in the IEEE 118-bus test system

The GS method fails to follow the conditions imposed by the PV bus if the power system size is large (Figure 2d and Figure 3). The results of the well-conditioned system show that the estimation of the initial values of the voltages is not far from the solutions found by the NR and RK4 methods. The proposed RK4 method always gives the best results compared to the other methods.

IV. Conclusion

This paper proposes the RK4 method based on the analogy between the NR method and the explicit Euler method in finding the solutions of PF. The PF problem formulation and associated algorithms were presented and applied to the IEEE standard 9-bus, 14-bus, 30-bus, 58-bus, and 118-bus test systems. The GS method converges very well for small power system. On the other hand, the performance of this method reduces dramatically with increasing system size. Thus, the NR method is very robust especially for large system size.

The simulation results of the RK4 method show that the computation time and the number of iterations to reach convergence increase with the size of system, but they are largely lower than those of the GS method; the convergence of the systems requires less number of iterations; and the number of iterations required for the convergence of system is constant when the computational accuracy increases. The convergence performance of the RK4 method is not better than that of the NR method. However, the robustness of both methods is the same, and that of the RK4 method is weaker. On the other hand, the performance of RK4 method is better than the performance of GS method.

This method proposed in this paper is very fast and reliable to use for solving the PF equations of large-scale well-conditioned systems. Future work would focus on using the combination of methods to reduce the number of inversion of the Jacobian matrix, especially, Runge-Kutta and Broyden methods.

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