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An Analysis of Functional Link Neural Networks (FLNN) Assisted Active Noise Cancelling Systems

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Abstract: *In this article, a study on Active Noise Control (ANC) systems assisted by Functional link neural networks (FLNN) is presented. Through analyzing the effects of nonlinearity in the components in ANC systems, different types of nonlinear distortions which are common in practical ANC systems are discovered. Furthermore, based on the ability to model the nonlinearity of FLNN, the effectiveness of ANC systems that utilize this model has been studied. Several quantitative simulations in different non-linear scenarios in ANC systems were conducted to prove the accuracy of analyses.*

Keywords: *Active Noise Control, Functional Link Neural Networks, Nonlinear Distortion, Adaptive Algorithms*

I. Introduction

Active noise control (ANC) has been acknowledged as one of the most effective methods to control noise, especially in low frequencies (no higher than 500Hz). However, this technology has truly become popular due to adaptive filtering techniques becoming more advanced and accessible. ANC systems using FIR filters and Filtered-X Least Mean Squares (Fx-LMS) algorithms have shown superior characteristics compared to earlier ANC systems [1-3]. To further improve noise cancellation, different adaptive algorithms for ANC systems based on linear control systems have been developed [4, 5]. However, in practical usage, reference noise can be nonlinear and components in ANC systems can also be affected by nonlinearity [6, 7]. Consequently, ANC systems based on linear control systems can be compromised.

To compensate for this drawback, many ANC systems based on nonlinear control systems (e.g., neural networks, LIP filters) have been proposed [6-10]. Among these nonlinear control systems, FLNN is widely utilized for its ability to scale effectively and relative simplicity. Many FLNN-based ANC systems and further structural improvements for these systems have been developed for real-world use cases [11-14]. In this study, we will analyze the effects of nonlinearity in ANC systems and evaluate the effectiveness of FLNN in these systems. In the first Section, analysis of nonlinearity on the primary path and, secondary path, and the source of noise in ANC systems will be detailed. Different types of nonlinearity (e.g., nonlinearity with memory, nonlinearity without memory, and chaotic nonlinearity) will be discussed. Which type of nonlinearity in ANC systems can be accounted for sufficiently or insufficiently by FLNN as well as the cause behind such relationship will be identified? Moreover, to defend these findings, comparisons between the noise-controlling characteristic of multiple control systems (i.e., FIR, SOV, FLNN) will be presented. FLNN-based ANC systems' ability to manage calculation resources will also be evaluated and compared.

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II. FLNN-based ANC systems

2.1. Struction

As discussed, FLNN was proposed as a simplistic replacement for Multi-layer Neural Networks (MLNN), as it has low computational complexity and can be easily deployed in real-world operations. Apart from the initial suggested application to identify references, FLNN has also been utilized in adaptive channel-balancing systems, to identify active nonlinear systems, and most notably, in nonlinear ANC systems. FLNN-based ANC systems were first introduced by Debi Prasad Das as illustrated in Figure 1.

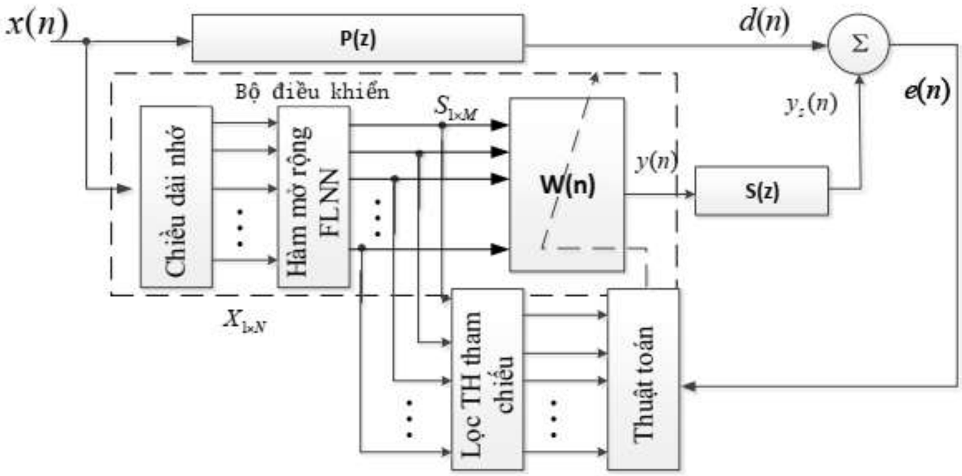


Figure 1. Block diagram of an FLNN-based ANC system utilizing Fx-LMS

Here, P(z) is the transfer function for the primary path (path from the reference microphone to the error sensor); S(z) is the transfer function of the secondary path (path from the output of the control system to the output of the error sensor); x(n) is the reference signal; y(n) is the output of the control system; ys(n) is the output of the secondary path, d(n) is the noise passing through the primary path; W(z) is the transfer function of the adaptive control system; S(z) is the estimate of the secondary path; v(n) is the filtered signal that has been passed from the expansion signal s(n) through S(z).

Assuming that X(n)=[x(n) x(n-1) … x(n-N+1)]T is the vector of the initial signal of the ANC system, the expansion P vector of this signal can be indicated through the FLNN function as follows:

$$\begin{aligned} S(n) = {} & [x(n), \sin(\pi\, x(n)), \cos(\pi\, x(n)), \ldots, \sin(P\pi\, x(n)), \cos(P\pi\, x(n)), x(n-1), \\ & \sin(\pi\, x(n-1)), \cos(\pi\, x(n-1)), \ldots, \sin(P\pi\, x(n-1)), \cos(P\pi\, x(n-1)) \ldots, \\ & x(n-N+1), \sin(\pi\, x(n-N+1)), \cos(\pi\, x(n-N+1)), \ldots, \\ & \sin(P\pi\, x(n-N+1)), \cos(P\pi\, x(n-N+1))]^T \end{aligned} \quad (1)$$

As illustrated in Figure 1, the output of control system y (n) was created by combining the weighted vector W (n) with the expansion signal S(n):

$$y(n) = W^T(n)\, S(n) \quad (2)$$

$W(n) = [w1, w2, \ldots, wM]^T$ *is the control system's adaptive weighted vector;* $[\bullet]^T$ *is the transpose; M=(2P+1).N*

2.2. Adaptive algorithms

In ANC systems, there exists a physical connection between the control system and the error sensor, which leads to instability when using common Least Mean Square (LMS) algorithms to update each filtered weight. To avoid this issue, the filtered-x least mean square algorithm (Fx-LMS) was developed by Burgress and company [15] and has been successfully applied in practical ANC use cases. In this study, we have employed this algorithm to update the filtered weights of W(n) in the control system using FLNN.

From the principle of ANC systems, we can identify that excessive noise e(n) that was detected by the error sensor is

$$e(n) = d(n) - A(n) * y(n) = d(n) - A(n) * [W^T(n)S(n)] \quad (3)$$

In which, A(n) indicates the impulse response of S(z) at n period and * indicates convolution. The purpose of the adaptive algorithm Fx-LMS in FLNN-based ANC systems is to reduce the cost function J(n) using gradient descent. From there, the cost function J(n) can be indicated with the functions of the capacity of excess noise, meaning,

$$J(n) = E(e^2(n)) \quad (4)$$

With E(.) indicating the expected value.
To minimize the cost function J(n), the weighted vector W(n) is updated according to the following equation:

$$W(n+1) = W(n) - \frac{1}{2}\mu\nabla_{W(n)}J(n) \quad (5)$$

Here, µ is the learning rate that helps adjust the converging rate of the algorithm; ∇W(n)J(n) is the gradient of the cost function J(n) relative to weighted vector W(n). With that, we have the following:

$$\nabla_{W(n)}J(n) = \frac{\partial J(n)}{\partial W(n)} \cong -2e(n)\frac{\partial y,(n)}{\partial W(n)} = -2e(n)\frac{\partial(A(n) * y(n))}{W(n)} = -2e(n)(A(n) * S(n)) \quad (6)$$

From (6) and (5),

$$W(n+1) = W(n) + \mu e(n)[a(n) * S(n)] \quad (7)$$

In a practical system, A(n) can be substituted by its weight. Hence, the weight updating algorithm of the filter in the FLNN control system can be rewritten as follows: $W(n+1)=W(n)+ \mu e(n)S_f(n)$

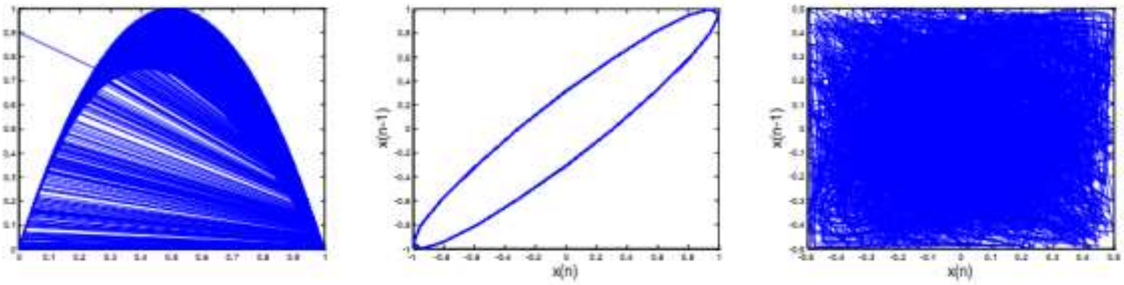
(8)

With Sf (n) = [A(n) * S(n)] being the filtered signal of S(n) through the secondary path.
Analysis of the nonlinearity of components in ANC systems

In reality, producing a comprehensive statement on the state of nonlinear systems is considerably more difficult compared to linear systems. With nonlinear systems, based on the superposition principle, there are powerful analytical methods such as the Laplace transform method and the Fourier method. From there, the adaptation of the systems can be indicated as one singular function, either a frequency adaptation or a pulse adaption. Nonlinear systems cannot be described as so. Different nonlinear systems function differently depending on the type of input. Therefore, it is illogical to attempt to describe their mechanic without identifying how they were triggered. Therefore, understanding the nonlinearities in specific cases can help precisely analyze their effects on the systems and provide appropriate models to characterize them

Many linear ANC systems have been widely used in controlling both narrow-band and wide-band noise [1-5]. Studies have theorized that the primary path, the secondary path, and the reference noise are linear. However, in practical situations, ANC systems may have to account for the nonlinearity of their components.

Firstly, we analyze the nonlinearity of reference noise. As reported in cited studies [16], noise from a mechanical system can usually be modeled like a chaotic nonlinear distortion, not a random process, timbre, or white noise. Particularly, it has been proven in [16] that the noise-inducing process in air conditioning systems is usually a chaotic behavior. Three main types of chaotic distortion are used as reference signals to evaluate the effectiveness of nonlinear ANC systems: Lorenz, Dufing, Logistic [6,16]. To demonstrate the dissimilarity of the reference signals, we have used Figure. 2 to illustrate Logistic distortion, monosyllabic distortion, and random distortion.



(a) (b) (c)

Figure 2. Phase graph of chaotic Logistic signals (a), monosyllabic signal (b), and random signals (c)

Secondly is the nonlinearity in the primary path. In many ANC applications (e.g., in jet turbines, engine sound reduction), due to the high pressure of the primary sound in the main tube, the primary noise can display nonlinearity at the noise reduction point. Evidently, in such cases, the nonlinearity of air is not required to be accounted for. As reported in studies [6,17], a typical scenario is a sine wave sound at 500 Hz traveling through a duct at L=140dB, creating about 1% of harmonic distortion after 1 Meter of travel. These distortions lead to the creation of pulse waves, and ANC systems based on linear control systems can hardly achieve sufficient noise control in such conditions.

Finally is the effects of nonlinearity on the secondary path. The effects of nonlinearity in the secondary path on ANC systems can be derived from two fundamental causes: Firstly, the secondary path's estimation from the output to the microphone controller can be erroneous, non-minimum phases can exist (i.e., the relationship between the system and its opposite is not causal) [6,16]; Secondly, the corrosion of electronic components can cause saturation distortion in the microphone, the signal converter, the pre-amp, and the power amplifier.

If one of these problems were to persist, linear ANC systems may not function properly and will show a significant reduction in their ability to control noise. Hence, when designing and deploying practical ANC systems, the effects of nonlinearity on ANC systems need to be addressed. Analysis of FLNN's ability to model nonlinearity for ANC systems

The nonlinear model of FLNN-based systems is solely related to the nonlinear point-wise extensions of the reference inputs at the same time. Particularly, if we assume *X(n)=[x(n), x(n-1), … ,x(n-N+1)]*^{T} as the vector of N components at the time *n*, expanding the nonlinear function of the first component gives the following result:

$$f_1[x(n)] = x(n); f_2[x(n)] = \sin[\pi x(n)]; f_3[x(n)] = \cos[\pi x(n)]; \ldots;$$
$$\ldots, f_{2P}[x(n)] = \sin[P\pi x(n)]; f_{2P+1}[x(n)] = \cos[P\pi x(n)] \quad (2\text{-}31)$$

Similarly, expanding the nonlinear function of the Nth component gives the result:

$$f_1[x(n-N+1)] = x(n-N+1); f_2[x(n-N+1)] = \sin[\pi x(n-N+1)]; f_3[x(n-N+1)] = \cos[\pi x(n-N+1)]$$
$$\ldots; f_{2P}[x(n-N+1)] = \sin[P\pi x(n-N+1)]; f_{2P+1}[x(n-N+1)] = \cos[P\pi x(n-N+1)] \quad (2\text{-}33)$$

The nonlinearity expansion function of FLNN is solely dependent on its immediate value. This means that it can model better in comparison to memory-less nonlinear systems. Therefore, FLNN-based ANC systems can achieve noise-controlling proficiency in the presence of nonlinear distortions like saturation.

The nonlinearity expansion function of FLNN does not contain its past input or output variables. For easy visualization, the basic expansion function of Volterra and FLNN can be compared. The expansion function of FLNN does not possess basic functions representing cross-rank numbers (x(n-i)mx(n-j)n, i≠j). In this scenario, FLNN may not effectively model nonlinear systems containing memory nonlinearity. In short, the noise-controlling characteristic of FLNN-based systems may be compromised in the presence of nonlinear distortion caused by latency (i.e., present state being dependent on past state).

In the scenario where the reference noise is a chaotic process, the characteristics of FLNN-based ANC systems will depend on the estimation of the secondary path. If the estimation of the secondary path contains a non-minimum phase, the characteristics of FLNN-based ANC systems will improve significantly compared to ANC systems based on linear control systems. As indicated by the studies in [18], linear control systems (e.g., FIR filters) are not suitable for predictions. However, FLNN filters are nonlinear filters that can closely estimate the causality of the secondary path with a non-minimum phase when input noise is created by a chaotic noise source.

III. Analysis of computational complexity

To evaluate the proficiency of ANC systems in reality, besides the ability to control noise, calculation cost is a crucial aspect. In this section, an analysis of the computational complexity of FLNN-based systems is presented. Assuming Ls is the memory size of the secondary path; N is the external input signal; P is the rank of the nonlinearity expansion functions. The computational complexity of FLNN-based calculation requires the following adjustment mathematical operations:

- Operations to calculate the output of the adaptive control systems: (2P+1)N multiplications and (2P+1)N-1 additions.
- Operations to update filter coefficients of the control system: (2P+1)N+1 multiplications and (2P+1)N additions.
- Operations to calculate the cost of filtering signals through the secondary path to update filter coefficients: (2P+1)Ls multiplications and (2P+1)(Ls-1) additions.

For a more general view, we compared the computational complexity of FLNN-based, FIR-based, and Volterra-based ANC systems and summarized the results in Table 1.

From Table 1, it can be seen that FLNN-based ANC systems have lower computational complexity than Volterra-based ANC systems. This can be a crucial advantage as the Volterra nonlinear filter has already been recognized as highly efficient in modeling nonlinear systems.

Table 1: The computational complexity of FIR-based, Volterra-based, and FLNN-based ANC systems, in addition to specific comparisons in simulated situations

FIR controller			Volterra controller		FLNN controller	
Operations	Add.	Multi.	Add.	Multi.	Add.	Multi.
Total	$2N+L_S-3$	$3N+L_S$	$13N+11(L_S-1)-2$	$19N+11L_S+5$	$(2P+1)(2N+L_S-1)-2$	$(2P+1)(3N+L_S)$
(N=10, P=2, Ls=5)	22	35	172	250	118	175

IV. Computer simulation

To prove the credibility of the aforementioned analyses, we compared FLNN-based ANC systems to FIR-based and Volterra-based ANC systems on their noise-canceling characteristic and their computational complexity.

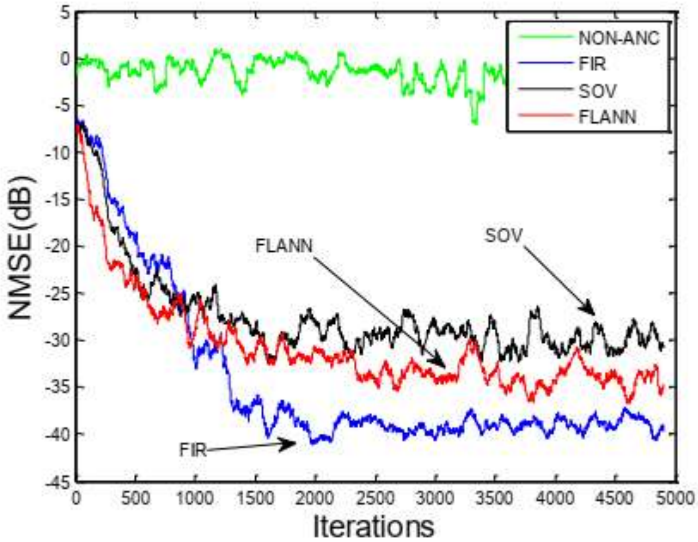


Figure 3a. Comparing the noise canceling of systems with chaotic reference signals on the secondary path with a minimum phase

Experiment 1: The reference signal is a logistic perturbation, created by recursion as explained in the following equation:

$$x(n+1) = \lambda x(n)[1 - x(n)] \quad (2\text{-}1)$$

With λ=4, x(0)=0.9, and n=1, 2, 3…. This process is then normalized to have a unit signal power to avoid saturation at the speaker.

The FIR linear filter is chosen as the transfer function of the primary path. The transfer function of the secondary path is also a linear filter but chosen in the following cases: a) The transfer function of the secondary path has a minimum phase; b) The transfer function of the secondary path has a non-minimum phase. Figure 3a shows that the efficiency of noise cancellation of ANC systems utilizing FLNN or Volterra control systems is lower compared to ANC systems using linear control systems. As discussed earlier, in this instance, the model can be regarded as identifying the linear system; the use of a nonlinear filter is unnecessary and will reduce the characteristic of the system. In Figure 3b, the efficiency of noise cancellation of the ANC system utilizing FLNN is the greatest. In this instance, FLNN can closely estimate under the law of cause and effect of the non-minimum phase secondary path better than Volterra and FIR.

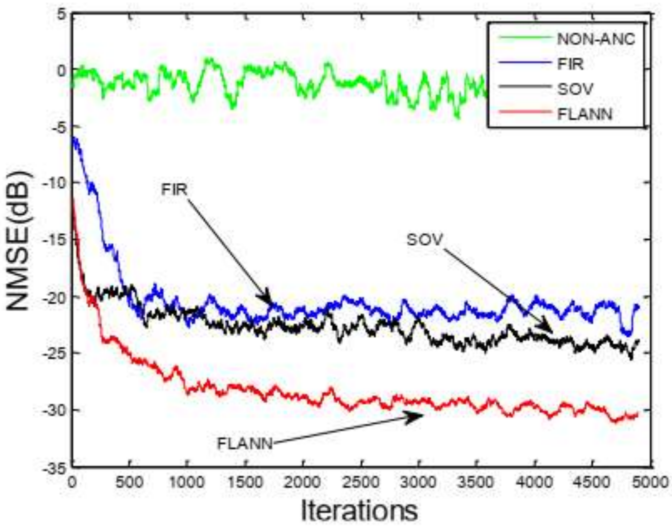


Figure 3b. Comparing the characteristics of control systems with reference signal and non-minimum phase secondary path.

Experiment 2: Primary path noise at the noise cancellation point is assumed to be affected by saturation. For instance, we can model it as the following cubic polynomial: $d(n) = x(n-2) + 0.08x^2(n-2) - 0.04x^3(n-2)$

(2-36)

Reference noise is a sine wave. The transfer function of the secondary path has a non-minimum phase. The results are shown in Figure 4. In the case of saturation, ANC systems utilizing FLNN or Volterra have far superior noise cancellation characteristics than systems utilizing FIR linear control systems. Furthermore, the characteristics of FLNN-based systems are better than SOV-based systems. As analyzed in earlier sections, saturated nonlinear systems are classified as a memoryless nonlinearity (the output is only dependent on the immediate value of the input). Moreover, by analyzing FLNN-based nonlinear models, we noticed that the expansion of its nonlinear function is only dependent on its immediate value. This means FLNN will form better models with systems containing memoryless nonlinearity.

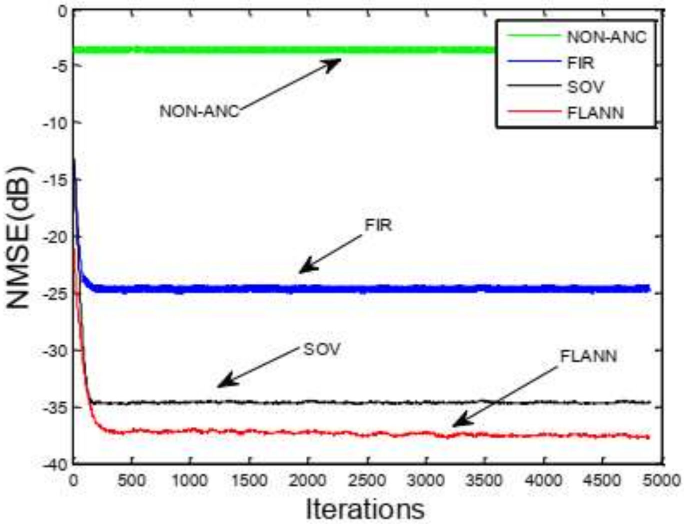


Figure 4: Comparing the characteristics of control systems containing memoryless nonlinearity

Experiment 3: Primary path noise at the noise cancellation point is assumed as a nonlinear deformation caused by latency. For instance, we can model the primary path and the secondary path with a corresponding input-output relationship as follows:

$$d(n) = x(n) + 0.8x(n-1) + 0.3x(n-2) + 0.4x(n-3) - 0.8x(n)x(n) + 0.9x(n)x(n-2) + 0.7x(n)x(n-3) \quad (2\text{-}37)$$

$$\hat{d}(n) = y(n) + 0.35y(n-1) + 0.09y(n-2) - 0.5y(n)y(n-1) + 0.4y(n)y(n-2) \quad (2\text{-}38)$$

A random noise is chosen as the reference signal. The simulation results are illustrated in Figure 5. It can be seen that noise cancellation characteristics achieved by FLNN-based ANC systems are significantly lower than Volterra-based ANC systems. As discussed earlier, the nonlinearity caused by this phenomenon is memory nonlinearity. Clearly, the output of this nonlinear system is dependent on not only the current value but also past input or output values. However, as demonstrated in Section 4, the nonlinearity expansion function of FLNN does not contain its input and/or output models. This means that FLNN lacks the basic functions representing cross-rank numbers (x(n-i)mx(n-j)n, i≠j). As a result, FLNN-based ANC systems cannot model nonlinear systems containing memory nonlinearity well.

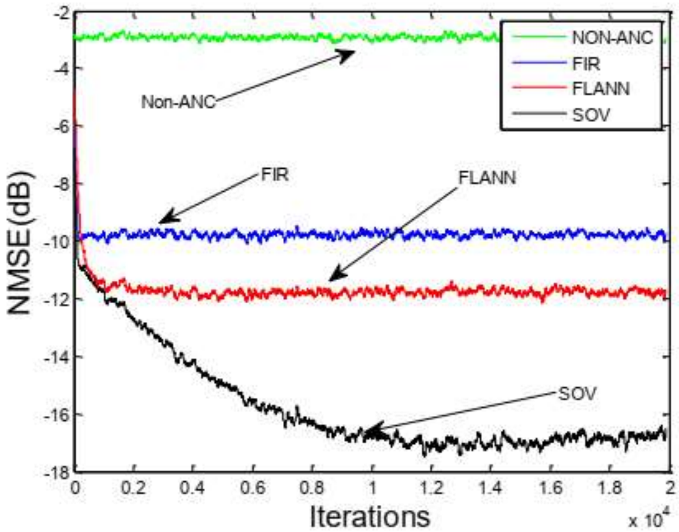


Figure 5: Comparing the characteristics of control systems containing memory nonlinearity

In short, it can be noted that FLNN-based ANC systems are appropriate in the following cases: First, saturated systems; Secondly, reference noise is a chaotic process and the secondary path has a non-minimum phase. When the system is affected by saturation with memory nonlinearity, FLNN-based ANC systems are unsuitable. Models based on the nonlinearity expansion of FLNN are unable to compensate for the nonlinear deformation caused by this phenomenon.

V. Conclusion

In this article, the nonlinear deformation in components of ANC systems and FLNN's ability to model has been studied. On that basis, we can understand the cause of nonlinear deformation, its intensity and the type of nonlinearity caused in components of ANC systems. Analysis of FLNN's ability to model shows that it is suitable for cases of memoryless nonlinearity and reference noise being a chaotic process. In the case of memory nonlinearity, its characteristics are reduced as the expansion function lacks cross-rank numbers. Comparing simulated results and computational complexity has proven the credibility and accuracy of the analysis.

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