



Modeling of Rainfall by the Fuzzy Logic Approach in the Boeny Region, Madagascar

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Abstract : The study area is the Boeny region, delimited in latitude between 15° South and 18° South and in longitude between 44° East and 48° East. The present study explores the ability of fuzzy logic in modelling rainfall in the Boeny region during the period 1979 to 2018. According to the fuzzy inference systems of order 2 and order 3, the models obtained fit better with the rainfall observation data. The low value of the average absolute error in percentage (less than 10%) confirmed that these models gave good results. For the year 2019, the average annual rainfall forecast is 43.5mm. Therefore, fuzzy logic offers an innovative new approach to rainfall modelling.

Keywords: Rainfall, Modelling, Fuzzy logic, Boeny region

I. Introduction

At the global level, the great threat facing scientists today is climate change. The African continent is the region of the world most vulnerable to climate change [1]. Some of the effects of climate change include rising sea levels, rising temperatures, ocean acidification, and changes in precipitation [2]. In Madagascar, the impact of climate change, in particular, rainfall remains a major concern for the Great Island. It is likely to hit the whole country hard in the next few years. Rainfall is one of the determining climatic factors in the climatic characterization of the different regions of Madagascar, especially the Boeny region. It is also the most important cause of the climate for both populations and ecosystems. It is an important indicator for studying climate change. And it is for this reason that this study leads us to model the average annual value of rain by the fuzzy logic method. In this perspective, the proposal of a model of rainfall processes and the forecast in a study area are essential steps to conduct this study.

II. Materials and Methods

1.1 Presentation of the study area

The study area (see Figure 1) is between latitude 15° South and 18° South and longitude between 44° East and 48° East.

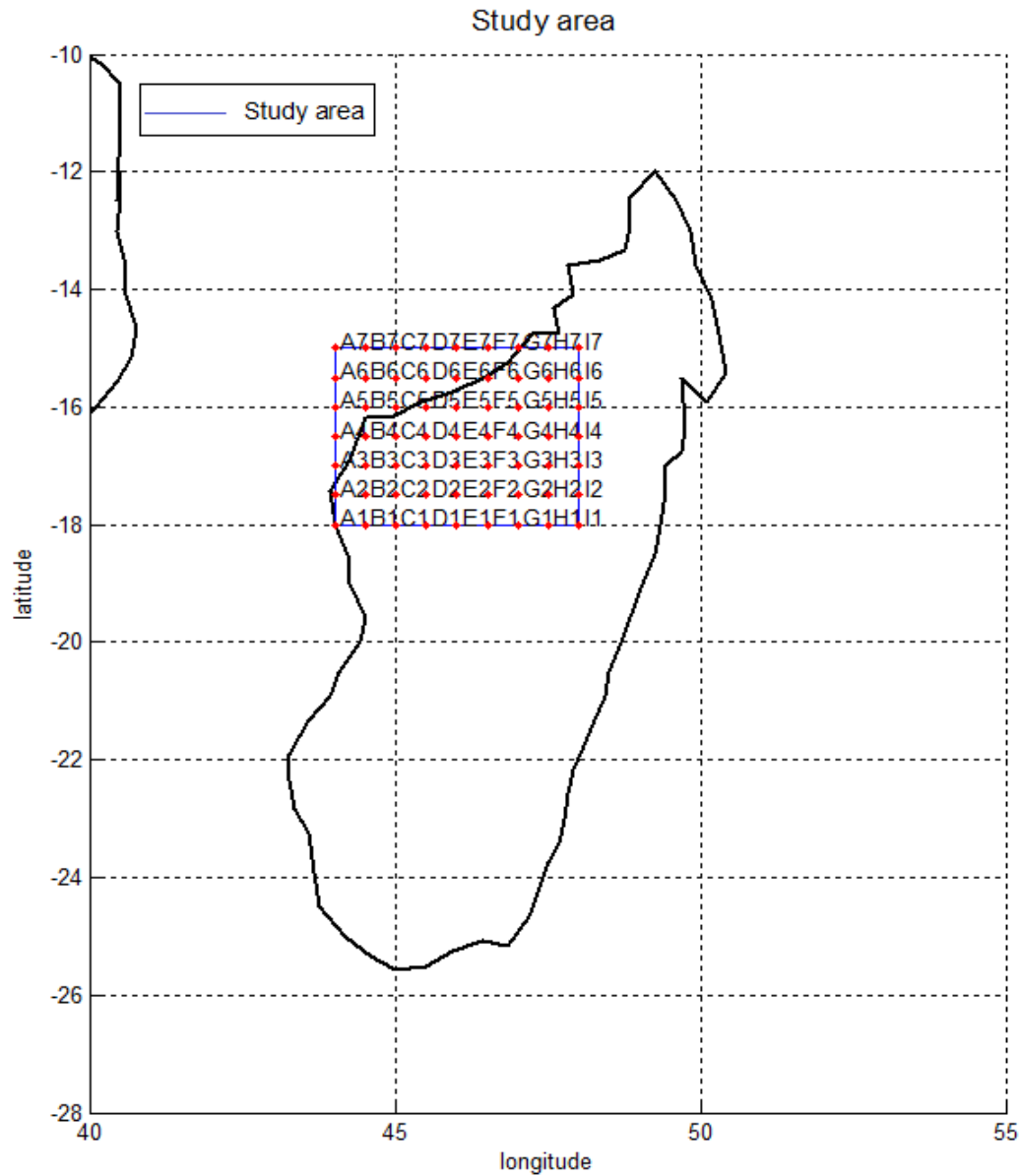


Figure 1 : Location of study area $44^{\circ} \leq \text{longitude} \leq 48^{\circ}$ et $-18^{\circ} \leq \text{latitude} \leq -15^{\circ}$.

In this area are Mahajanga I, Mahajanga II, Marovoay, Ambato Boeny, Soalala, Mitsinjo, Andanivato, Antanandava, Anosibe, Vilandrano, Ambodimanga, Vilamatsa, Bevatry, Bokarano, Mavozaza and Sambaokofa (Figure 2).



Figure 2 : Geographical location of the study area (Ancarta)

1.2 Database

The meteorological data that we used come from the daily reanalysis data of the experiment (ERA5) of the European Center for Medium range Weather Forecasts (ECMWF) at the synoptic scale with a grid of $0.5^\circ \times 0, 5^\circ$ rain over a temporal depth covering the period 1979-2018.

1.3 Fuzzy systems methodology

1.3.1 Fuzzy subsets

Fuzzy subsets were introduced in order to model the human representation of knowledge, and thus improve the performance of decision-making systems using modeling [3]. A fuzzy subset A defined on a universe of discourse U , is characterized by a membership function μ_A . An element x belongs to a subset A , with a membership degree $\mu_A(x)$ between 0 and 1.

1.3.2 Linguistic variable

Reasoning from imperfectly defined knowledge uses fuzzy logic to overcome the shortcomings of classical logic [4]. A (fuzzy) linguistic variable is therefore a variable whose fuzzy values belong to fuzzy sets that can represent natural language words. Thus a fuzzy variable can simultaneously take several linguistic values [5]. The linguistic variable X can be characterized by a triplet $(X, T(X), U)$, in which X is the name of the linguistic variable, $T(X)$ the set of linguistic values of X and U the universe of discourse [6]. Generally fuzzy logic uses the following rule: IF X is A , then Y is B .

1.3.3 Fuzzy inference system

A fuzzy inference system (FIS) offers a modeling approach very close to human reasoning to deal with imprecision and uncertainty. It can be considered logical systems that also uses linguistic rules to establish relationships between input and output variables [7]. The inputs come from the fuzzification process and the set of rules normally are defined by the know-how of the expert [8]. A fuzzy inference system is formed of three steps as shown in Figure 3. The first, fuzzification transforms the numerical values into degrees of membership of the different fuzzy sets of the partition. The second step is the inference engine, made up of

the set of rules. Finally, defuzzification is a decision-making step, which makes it possible to transform a fuzzy value of a variable into a real (net) value from the result of the aggregation of the rules. (Madami or sugeno).

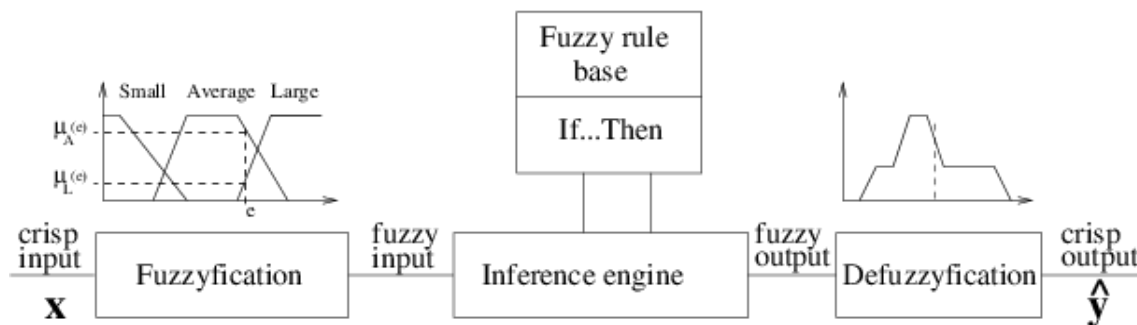


Figure 3: Fuzzy inference system (Guillaume 2005)

III. Results and Discussion

2.1 Learning parameters

2.1.1 Universe of discourse

The climatic variables to be modeled are time series of annual average precipitation from 1979 to 2018 during the 40 years of study. These dates are used as the input and output of the SIF model. In this study, the universe of discourse, the number of partitions and the number of entries for the rain data are given by Table 1:

Table 1: Universe of discourse, number of partitions and number of rain entries

Annual averages	Universe of discourse	Number of partitions	Input Number
Rain	U2= [22,29 48,18]	60 partitions (A1, A2, ... A60)	Two entries
Rain	U3= [22,29 48,18]	60 partitions (a1, a2, ... a60)	Three entries

2.1.2 Membership function

The membership function can be represented as a triangular, trapezoidal, parabolic, Gaussian, sigmoid, etc. function. For the sake of clarity and to facilitate the calculations, we have used the membership function of triangular type. The following figure presents the rain membership function.

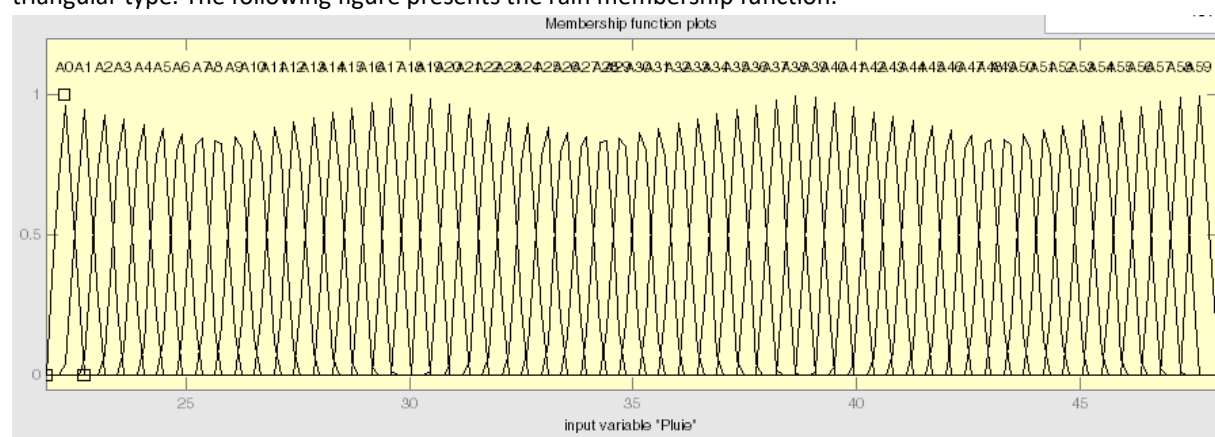


Figure 4: Membership functions of the annual average rainfall

2.2 Implementation of a SIF for annual average rainfall values

The realization of a fuzzy inference system goes through several steps: the fuzzification and defuzzification of the input and output variables, and the realization of an inference engine. The structure of a SIF for modeling

the average rainfall is illustrated in Figures 5 and 6. There are several approaches for the fuzzy inference system. In general, all approaches can be applied in fuzzy systems. In our case, we exploited the SIF model of the Mamdani type (1974).

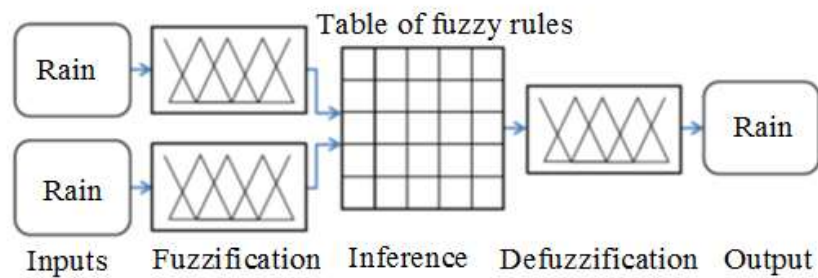


Figure 5: Structure of a SIF for the average annual rainfall with two inputs

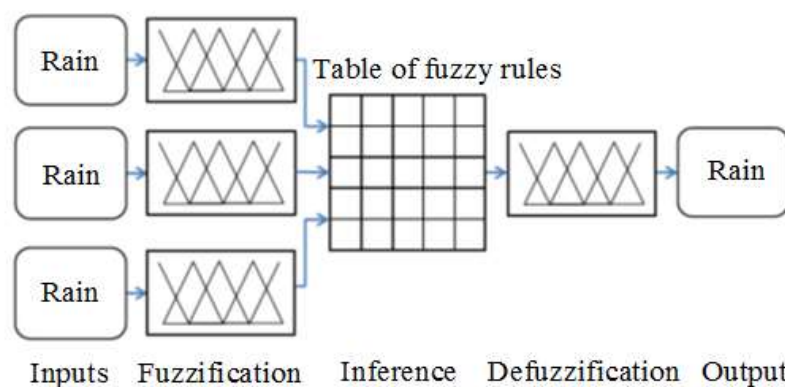


Figure 6: Structure of a SIF for the average annual rainfall with three inputs

2.2.1 Fuzzification

This step allows the transformation of the physical quantities of the climatic parameters into linguistic variables. Table 2 summarizes the numerical values as well as the values transformed into linguistic terms of the average annual rainfall during the study period (1979-2018).

Table 2: Fuzzification of annual mean value of rainfall

YEAR	REAL	FUZZY WITH TWO INPUTS			FUZZY WITH THREE INPUTS		
1979	33.02	A25			A25		
1980	35.41	A31			A31		
1981	35.18	A30	A25, A31	35,2	A30		
1982	42.42	A47	A31, A30	42,5	A47	A25, A31, A30	42,5
1983	41.81	A45	A30, A47	41,6	A45	A31, A30, A47	41,8
1984	40.50	A42	A47, A45	40,3	A42	A30, A47, A45	40,5
1985	27.97	A13	A45, A42	27,9	A13	A47, A45, A42	27,9
1986	32.40	A24	A42, A13	32,6	A24	A45, A42, A13	32,6
1987	28.78	A15	A13, A24	28,7	A15	A42, A13, A24	28,7
1988	32.43	A24	A24, A15	32,6	A24	A13, A24, A15	32,6
1989	34.78	A29	A15, A24	34,8	A29	A24, A15, A24	34,8
1990	26.25	A9	A24, A29	26,1	A9	A15, A24, A29	26,1
1991	36.59	A33	A29, A9	36,5	A33	A24, A29, A9	36,5
1992	32.55	A24	A9, A33	32,6	A24	A29, A9, A33	32,6

1993	41.87	A46	A33, A24	42,1	A46	A9, A33, A24	41,9
1994	37.71	A36	A24, A46	37,8	A36	A33, A24, A46	37,8
1995	36.29	A33	A46, A36	36,5	A33	A24, A46, A36	36,5
1996	35.79	A31	A36, A33	35,6	A31	A46, A36, A33	35,8
1997	40.02	A41	A33, A31	39,9	A41	A36, A33, A31	39,9
1998	39.59	A40	A31, A41	39,5	A40	A33, A31, A41	39,6
1999	30.01	A18	A41, A40	30	A18	A31, A41, A40	30
2000	33.22	A25	A40, A18	33,1	A25	A41, A40, A18	33,1
2001	31.78	A22	A18, A25	31,8	A22	A40, A18, A25	31,8
2002	29.83	A18	A25, A22	30	A18	A18, A25, A22	30
2003	34.69	A29	A22, A18	34,8	A29	A25, A22, A18	34,8
2004	43.80	A50	A18, A29	43,8	A50	A22, A18, A29	43,8
2005	27.28	A12	A29, A50	27,4	A12	A18, A29, A50	27,4
2006	24.76	A6	A50, A12	24,9	A6	A29, A50, A12	24,9
2007	43.46	A49	A12, A6	43,4	A49	A50, A12, A6	43,4
2008	37.54	A35	A6, A49	37,3	A35	A12, A6, A49	37,5
2009	30.43	A19	A49, A35	30,5	A19	A6, A49, A35	30,5
2010	36.09	A32	A35, A19	36	A32	A49, A35, A19	36
2011	37.39	A35	A19, A32	37,3	A35	A35, A19, A32	37,3
2012	41.39	A44	A32, A35	41,2	A44	A19, A32, A35	41,3
2013	30.64	A19	A35, A44	32	A19	A32, A35, A44	30,5
2014	37.96	A36	A44, A19	37,8	A36	A35, A44, A19	37,8
2015	41.07	A44	A19, A36	41,2	A44	A44, A19, A36	41,2
2016	38.29	A37	A36, A44	38,2	A37	A19, A36, A44	38,2
2017	35.68	A31	A44, A37	35,6	A31	A36, A44, A37	35,6
2018	35.81	A31	A37, A31	35,6	A31	A44, A37, A31	35,8

2.2.2 Creating fuzzy rules

3.2.2.1 Inference process

For rainfall modeling, after several scenarios and several tests, we fixed a second and third order SIF model. These are models for a two-input output and for a three-input output. Table 3 presents the fuzzy rules adopted. The numbered letters are linguistic variables used to model rainfall during the study period.

Table 3: Example of fuzzy rules for rain model of order 2 and order 3

Rain model (order 2)	Rain model (order 3)
Previous ---> consequent	Previous ---> consequent
A25, A31 ---> A30	A25, A31, A30 ---> A47
A31, A30 ---> A47	A31, A30, A47 ---> A45
A30, A47 ---> A45	A30, A47, A45 ---> A42
.....	
.....	
A37, A31 ---> A31	A44, A37, A31 ---> A31

3.2.3 Defuzzification

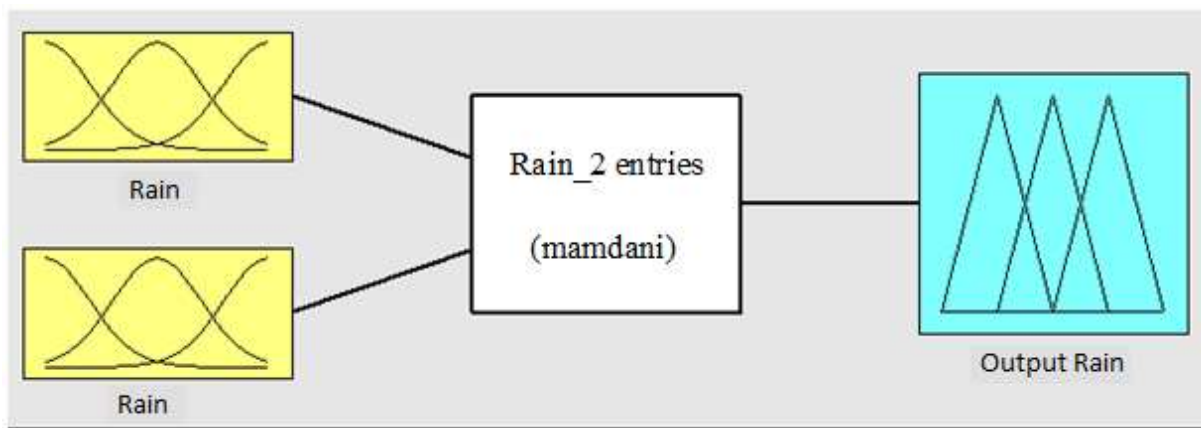
The input for the defuzzification process is the combinatorial result of the fuzzified set. The objective is to transform this fuzzy set into non-fuzzy values [9]. The table 4 shows the defuzzified values of the rainfall model output.

Table 4: Average rainfall defuzzification

YEAR	REAL [mm]	OUTPUT VALUES [mm]		YEAR	REAL [mm]	OUTPUT VALUES [mm]	
		ORDER 2	ORDER 3			ORDER 2	ORDER 3
1979	33.02			1999	30.01	30	30
1980	35.41			2000	33.22	33,1	33,1
1981	35.18	35,2		2001	31.78	31,8	31,8
1982	42.42	42,5	42,5	2002	29.83	30	30
1983	41.81	41,6	41,8	2003	34.69	34,8	34,8
1984	40.50	40,3	40,5	2004	43.80	43,8	43,8
1985	27.97	27,9	27,9	2005	27.28	27,4	27,4
1986	32.40	32,6	32,6	2006	24.76	24,9	24,9
1987	28.78	28,7	28,7	2007	43.46	43,4	43,4
1988	32.43	32,6	32,6	2008	37.54	37,3	37,5
1989	34.78	34,8	34,8	2009	30.43	30,5	30,5
1990	26.25	26,1	26,1	2010	36.09	36	36
1991	36.59	36,5	36,5	2011	37.39	37,3	37,3
1992	32.55	32,6	32,6	2012	41.39	41,2	41,3
1993	41.87	42,1	41,9	2013	30.64	32	30,5
1994	37.71	37,8	37,8	2014	37.96	37,8	37,8
1995	36.29	36,5	36,5	2015	41.07	41,2	41,2
1996	35.79	35,6	35,8	2016	38.29	38,2	38,2
1997	40.02	39,9	39,9	2017	35.68	35,6	35,6
1998	39.59	39,5	39,6	2018	35.81	35,6	35,8

2.2.3 Mamdani SIF model by matlab software

The structure of the SIF Mamdani model obtained by the Matlab software is presented in figure 7. One of this model is formed by 2 inputs, an output with 60 fuzzy rules and the other is formed by 3 inputs, an output always with 60 fuzzy rules.



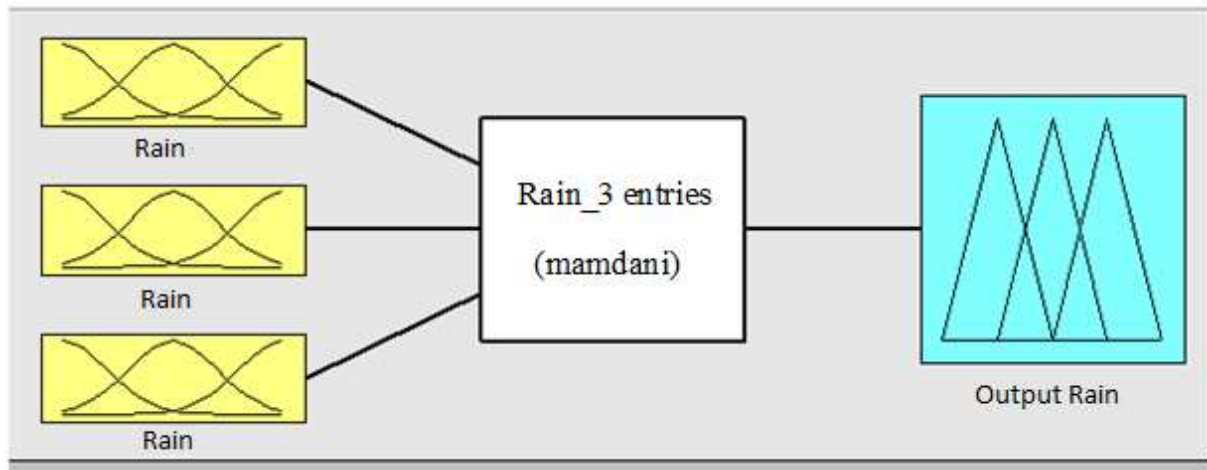


Figure 7: Fuzzy logic models in Matlab

2.2.4 Graphical representation of the model

Figures 8 and 9 show the time series of rainfall forecasts observed during the study period and respectively for the SIF models. We note that the curves of the observation data (in black) are confused with that of the models obtained (in blue). This suggests that we have good models. For the short-term forecast, the rainfall height value for the year 2019 is 43.5mm for the two-order model and the three-order model respectively.

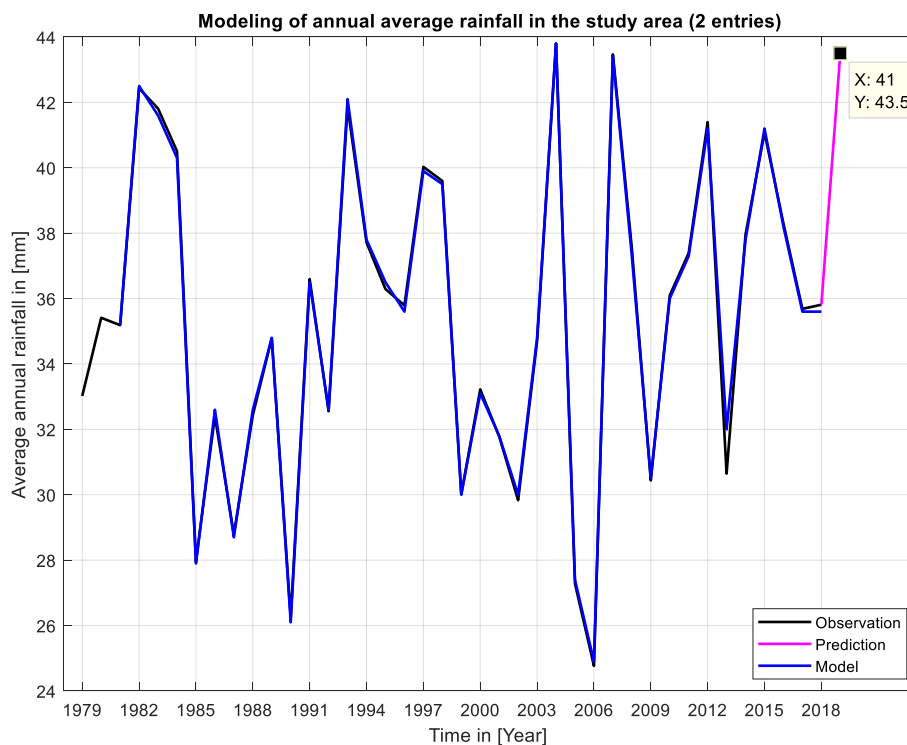


Figure 8: Two-input rainfall forecast model curve

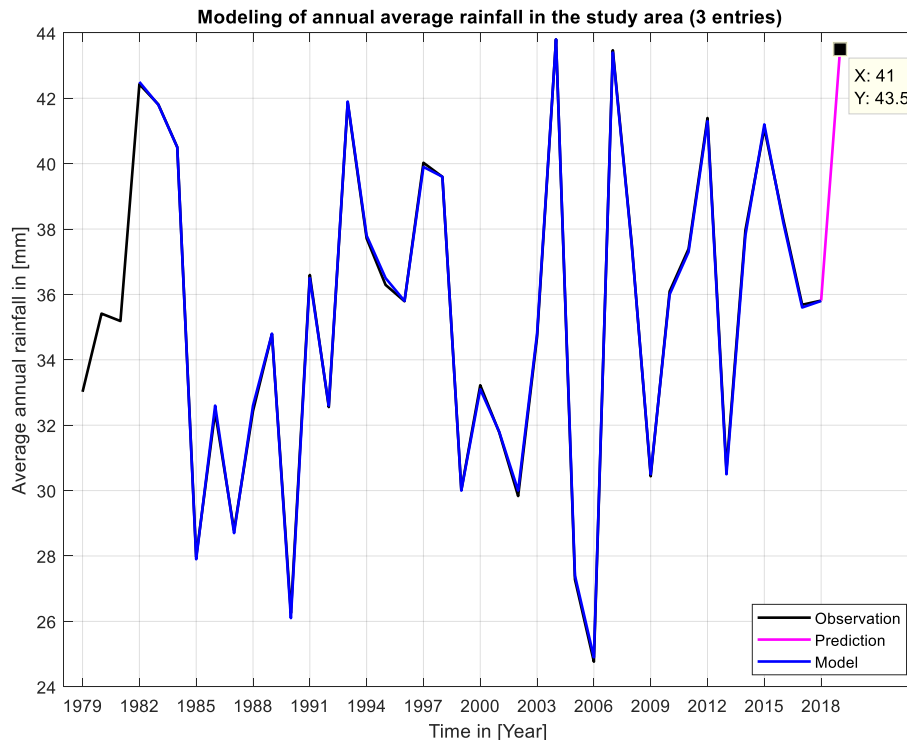


Figure 9: Three-input rainfall forecast model curve

2.2.5 Model validation criteria

The performance measures of SIF models are mean absolute percentage error (MAPE) and percent accuracy (P). The calculated MAPE values as well as the accuracy percentages are summarized in Table 5. The average absolute error in percentage is successively 0.44% for the second-order model and 0.25% for the three-order model. Since the MAPE are very low, the SIF model gives satisfactory results. The percentage of accuracy is respectively 99.92% (order two) and 99.99% (order three). However, the accuracy of the results is very high. Indeed, the simulation results show that our models retained for the determination of rainfall forecast are excellent.

Table 5: Values of MAPE and percent accuracy of the model

Validation criteria	FUZZY WITH TWO INPUTS	FUZZY WITH THREE INPUTS
MAPE	0,44 %	0,25 %
P	99,92%	99,99%

IV. Conclusion

This study proposes a fuzzy logic method for modeling the average annual rainfall from 1979 to 2018 in the Boeny region of Madagascar. The models retained for the average rainfall values are of order 2 and order 3 with 60 fuzzy rules (partitions). The fuzzy inference system models used fit better for rainfall observation data. According to the MAPE validation criterion, both models receive a percentage lower than 1%. The accuracy of the models is very high. As well as the average annual rainfall value for the year 2019 is 43.5mm. Finally, it would be interesting to use hybrid models such as nero-fuzziness or another method to determine the medium-term rainfall forecast.

V. References

- [1] H.-O. Pörtner, D.C. Roberts, E.S. Poloczanska, K. Mintenbeck, M. Tignor, A. Alegría, M. Craig, S. Langsdorf, S. Löschke, V. Möller, A. Okem (eds.)). In: *Climate Change 2022: Impacts, Adaptation, and Vulnerability*. Cambridge University Press. In Press.
- [2] P.R. Shukla, J. Skea, E. Calvo Buendia, V. Masson-Delmotte, H.-O. Pörtner, D. C. Roberts, P. Zhai, R. Slade, S. Connors, R. van Diemen, M. Ferrat, E. Haughey, S. Luz, S. Neogi, M. Pathak, J. Petzold, J. Portugal Pereira, P. Vyas, E. Huntley, K. Kissick, M. Belkacemi, J. Malley, IPCC, 2019: Climate Change and Land: an IPCC special report on climate change, desertification, land degradation, sustainable land management, food security, and greenhouse gas fluxes in terrestrial ecosystems. In press.
- [3] P. A. Bisgambiglia (2008). *Approche de modélisation approximative pour des systèmes à événements discrets: Application à l'étude de propagation de feux de forêt* (Doctoral dissertation, Université Pascal Paoli).
- [4] L. Zadeh, (1965). Fuzzy sets. *Information and Control*, 8(3):338 – 353.
- [5] C. C. Lee, "Fuzzy logic in control systems: fuzzy logic controller. Part1", *IEEE Transactions on systems Man and Cybernetics*, Vol. 20, 1990, pp. 404-418.
- [6] Ralescu, A., & Hartani, R. (1995, March). Some issues in fuzzy and linguistic modeling. In *Proceedings of 1995 IEEE International Conference on Fuzzy Systems*. (Vol. 4, pp. 1903-1910). IEEE.
- [7] Talbi, N. (2019). Design of fuzzy controller rule base using bat algorithm. *Energy Procedia*, 162, 241-250.
- [8] E. Bonilla-Huerta, B. Duval and J. K. Hao. Fuzzy logic for elimination of redundant information of microarray data. *Genomics Proteomics and Bioinformatics*, Elsevier, Octobre 2008, Vol.6, No. 2
- [9] S. Zribi Boujelbene, D. Ben Ayed MezghaniN. Ellouze. Systèmes à Inférences Floues pour la Classification Phonémique. SETIT 07 4th International Conference: Sciences of Electronic, Technologies of Information and Telecommunications, TUNISIA, March 25-29, 2007