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Dividing a Sphere into Rings and Polar Lunes for Evaluating the Factor Q of Directivity of a Loudspeaker

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ABSTRACT: *Before the use of acoustic prediction software to design rooms, practitioners generally used several formulas that always required a called Directivity Factor (Q) of the loudspeakers to determine the directivity of a sound source. To calculate the Factor Q of a loudspeaker this must be assumed to be inside an imaginary sphere with a unit-surface that can be divided into rings or polar lunes. Then, with the on-axis and the average SPL around the sphere solve the Q formula. In this paper, the authors present both methods of division of the sphere and use one of them to get the Factor Q of a loudspeaker system. Afterward, compared the Q result obtained with the Q Factor from the manufacturer's datasheet of the system.*

I. INTRODUCTION

The Factor Q is defined as the ratio between the on-axis Sound Pressured Level (SPL) and the average of the SPL over a sphere of a surface equal to the unity around the sound source.

Hopkins and Stryker, in their seminal paper of 1948, defined the Directionality Index [1]. They followed a similar procedure used before by Molloy of dividing the sphere into rings and polar lunes [2]. In his research Molloy determined that the ring method is the best for cone loudspeakers or symmetrical one-cell horns and that the polar lunes are best for all other types of sound sources. In 1973, Don Davis proposed a more elaborated methodology to measure the Factor Q, based on the division of the spere of Hopkins and Stryker, which was somewhat different than the one previously used by the manufactures [3]. However, G.L. Wilson claims that there are differences between his methods and the proposed by David's due to the difficulty of representing in a two-dimensional drawing the geometry of a spherical surface [4].

We wanted to calculate the Factor Q of a JBL™ compact loudspeaker system, Vertec™ Line Array 4886 [2] on a given fixed low frequency. (See figure 1). Working with a low frequency allows us to compare the efficiency of the Davis' Method because, at low frequencies, the accuracy factor Q is more difficult to find. Decreasing the frequency will decrease the directionality of the source. We would like to compare the Davis' Method results with the value o the manufacturer specification Chart, adjusting the distances, because the JBL chart was made at 4 meters meanwhile ours was made at 1 meter.

F

Figure 1 System to be tested

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Measurement Procedure

The System was suspended one meter from the floor (See Figure 2). The microphone used was an Earthwork™ The gatherings were done according to Don Davis' Method around the array, taking the Sound Pressure level at the different angles described in the tables, are included on-axis one.

Figure 2 Preparing the system

Don Davis' Method

The so-called Equal Angle, Weighted-Area Method is derived out of work done by Ben Bauer, C.T. Molloy, and Bob Beavers [3]

"Starting at the 0° on-axis point of the polar plot, assign an arbitrary value of 100dB to the 0° point. Tabulate the relative differences in level, referred to this level, for each 10° point at all the ways around the horizontal plot. Continue on the vertical plot in the same manner but skipping the 0° and 180° points (Already recorded. Convert each Lp level to a relative power ratio (Rel Lp). Multiplying the ratio by a weighting factor proportional to the area surrounding the measuring points in terms of a sphere with surface of a unity. Total all the weighted power ratios (Lpw). Then subtract 10 times de logarithm of the sum from the on-axis reading of 100dB and take the power antilog."

II. DIVIDING THE SPHERE INTO RINGS.

The graph shown below presents a sphere divided into rings (Figure 3) and a section of the sphere (Figure 4). From the latter we obtain the following trigonometric relationship:

$b = r * \sin\theta$[5]

Figure 3 Sphere divide into rings

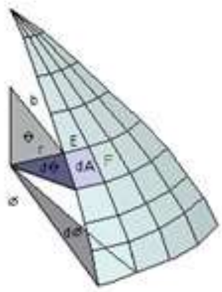

Figure 4 Spherical surface

Getting one side of the differential *dA* of the section of the sphere

If we multiply the above equation by the differential of the horizontal angle where $b = r * sin\theta$ we will obtain E which is one of the required sides of the Area Differential *dA* as shown next.

$E = b * d$

$E = r * \sin\theta * d\emptyset$ (I)

A subsection of the sphere is reproduced here for convenience of the reader and ease of explanation. (Figure 5)

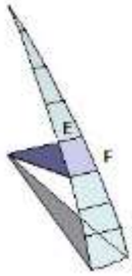

Figure 5 Area Differential

1.1.2. Determining, F, of the differential *dA*

To find F from the above figure and following an analogous process as before, we can deduce that

$$F = r * d\theta$$

Also, from this figure we can determine the area *dA* as

$$dA = E * F$$

By replacing the values of E from formula (I) into the last equation we obtain

$$dA = r^2 * \sin\theta * d\theta * d\emptyset$$

1.2. Finding the Total Area of the Sphere

By integrating the latter equation, it is possible to obtain the total area of all *dAs*. Accordingly,

$$\int_0^{\emptyset}\int_{\theta 1}^{\theta 2} r^2 * \sin\theta * d\theta * d\emptyset$$

Solving the integral for *dϑ* for one hemisphere

$$\int_0^{2\pi} (r^2 * \cos\theta \,\Big|_{\theta 1}^{\theta 2})\, d\emptyset$$

$$r^2 \int_0^{2\pi} (\cos\theta 1 - \cos\theta 2) * d\emptyset$$

$$r^2 * \emptyset * (\cos\theta 1 - \cos\theta 2)\Big|_0^{2\pi}$$

$A = 2\pi * r^2 * (\cos\theta 1 - \cos\theta 2)$ (II)
Observe that to avoid a negative value we have changed the order of the upper and lower limits of the definite integral. The resulting area is equal to $4\pi * r^2$.

Using the formula indicated by [6] we know that the sphere area is

$$A = 4\pi * r^2$$

Now, if the area is A=1, the above formula becomes

$$A = 1 = 4\pi * r^2$$

Accordingly, the radius *r* of a Sphere of area A=1 would be $r = \sqrt{\frac{1}{4\pi}}$

Therefore, for a hemisphere of a sphere of area A=1 we will obtain

$$\frac{A}{2} = 1 = 2\pi * r^2$$

With a radius $r = \sqrt{\frac{1}{2\pi}}$

Substituting this value of the radius into the formula (II) the hemisphere area we have

$$Ah = 2\pi * r^2 * (\cos\theta 1 - \cos\theta 2)$$

But $r = \sqrt{\frac{1}{2\pi}}$ for a hemisphere of Area A=1, so

$$Ah = 2\pi * \left(\sqrt{\frac{1}{2\pi}}\right)^2 * (\cos\theta 1 - \cos\theta 2)$$

Where we finally obtain,
$Ah = (\cos\theta 1 - \cos\theta 2)$ (III)

1.3.-Evaluation

With formula (III) we can now calculate the ring areas at 10° interval for the given hemisphere. Working in each ring, except in the first and last rows, we get.

Interval	Limits	Ring Area
0°	0°~ 15°	0.003805302
10°	5°~ 15°	0.030268872
20°	15°~ 25°	0.059618039
30°	25°~ 30°	0.087155743
40°	35°~ 45°	0.112045263
50°	45°~ 55°	0.133530345
60°	55°~ 65°	0.150958175
70°	65°~ 75°	0.163799217
80°	75°~ 85°	0.171663302
90°	85°~90°	0.087155743
	Total Area	1.000000000

Table 1 Ring areas

1.4. Plotting Sphere in rings

Based on the procedure of reference No. 5, we must now change only the radius ratio so that will have a sphere with A=1.

$$A = 2\pi * r^2 * (\cos\theta 1 - \cos\theta 2)$$

We know also that the radius r can be expressed as

$$r = \sqrt{\frac{1}{4\pi}}$$

Therefore, substituting in the general equation (II) mentioned before and simplifying the operation we obtain

$$A = 2\pi * \left(\sqrt{\frac{1}{4\pi}}\right)^2 * (\cos\theta 1 - \cos\theta 2)$$

$$A = \frac{1}{2}(\cos\theta 1 - \cos\theta 2)$$

Therefore, to find an area between 10° and 350° we use θ_1= 5° and θ_2= 345°. Applying the formula we will have

$$A = \frac{1}{2} * (\cos 5° - \cos 345°)$$

$$A = 0.015134436$$

This is the area of the ring located to 5° from the front axis of the loudspeaker. The ponderation factor proposed by Don Davis [7] in the Step 4 on his method (not shown on this paper) asks for an 8-sample test uniformly distributed over the Surface of each ring. These are 2 horizontals, 2 verticals and 4 diagonals. If the areas obtained in the previous table are divided by 8 we obtain the following table. The angles ratio on the first column are not shown in Don Davis' work.

Intervals in the Book	Formula Limits[8]	1/8 de area Except axial	Partial Sub-areas
0°	0°~ 350°		0.001902651
10°~ 350°	05°~ 345°	8*0.001891804	0.015134436
20°~ 340°	15°~ 335°	8*0.003726127	0.029809020
30°~ 350°	25°~ 325°	8*0.005447234	0.043577871
40°~ 320°	35°~ 315°	8~0.007002829	0.056022632
50°~ 310°	45°~ 305°	8*0.008345647	0.066765172
60°~ 300°	55°~ 295°	8*0.009434886	0.075479087
70°~ 290°	65°~ 285°	8*0.010237451	0.081899608
80°~ 280°	75°~ 275°	8*0.010728956	0.085831651
90°~ 270°	85°~ 265°	8*0.010894468	0.087155743
100°~ 260°	95°~ 255°	8*0.010728956	0.085831651
110°~ 250°	105°~ 245°	8*0.010237451	0.081899608
120°~ 240°	115°~ 235°	8*0.009434886	0.075479087
130°~ 230°	125°~ 225°	8*0.008345647	0.066765172
140°~ 220°	135°~ 215°	8*0.007002829	0.056022632
150°~ 210°	145°~ 205°	8*0.005447234	0.043577871
160°~ 200°	155°~ 195°	8*0.003726127	0.029809020
170°~ 190°	165°~ 185°	8*0.001891804	0.015134436
180°			0.001902651
			1.000000001

Table 2 Sample test

III. DIVIDING A SPHERE INTO POLAR LUNES.

In the work Calculating the Directivity Index for various types of radiators by C.T. Molloy [2], the author proposes to use the limits of two planes through the X axis making an angle of β with each other and two planes at angle α passing through the Z Axis. According to Molloy [2], the area intercepted by these two planes can be written as follow (Figure 6).

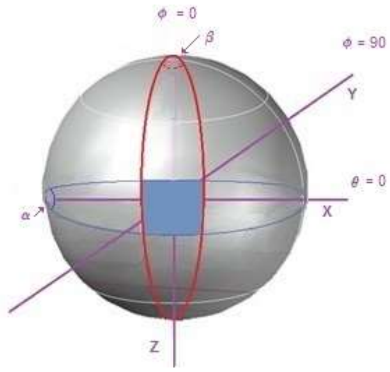

Figure 6 Sphere divide into polar lunes

$$A = 4 * \int_{\emptyset=\frac{\pi}{2}-\frac{\alpha}{2}}^{\frac{\pi}{2}} \int_{\theta=\eta}^{\frac{\pi}{2}} r * \sin\theta * d\phi * r * d\theta$$

From Reference No. 4, we have that $\eta = \cot^{-1}\left(\frac{r * \sin\phi * \sin\frac{\beta}{2}}{r * \cos\frac{\beta}{2}}\right)$

Simplifying the expression and using the definition of tangent of an angle we have that η can be expressed as

Also $\eta = \cot^{-1}\left(\sin(\emptyset * \tan(\frac{\beta}{2})\right)$ from where, according to reference [2], the result of the integral is

$$A = 4r^2 * \sin^{-1}\left(\sin\frac{\alpha}{2} * \sin\frac{\beta}{2}\right)$$

Knowing that $r = \left(\sqrt{\frac{1}{4\pi}}\right)^2$ and substituting in the last equation we obtain that

$$A = \frac{1}{\pi} * \sin^{-1}\left(\sin\frac{\alpha}{2} * \sin\frac{\beta}{2}\right)$$

2.1.-Evaluation of Results

Following a procedure like the one on Section 1.3 at 10° we obtain that (See Table 3)

$$A = 2 * \left(\frac{1}{\pi} * \sin^{-1}\left(\sin\frac{10}{2} * \sin\frac{350}{2}\right)\right) = 0.004835888$$

$$A = 2 * \left(\left(\frac{1}{\pi} * \sin^{-1}\left(\sin\frac{30}{2} * \sin\frac{330}{2}\right)\right) - 0.00485888\right) = 0.037841509$$

$$A = 2 * \left(\left(\frac{1}{\pi} * \sin^{-1}\left(\sin\frac{50}{2} * \sin\frac{310}{2}\right)\right) - 0.00485888 - 0.037841509\right) = 0.071640216$$

Intervals	Limits	Quadrangle Area
0°	10°~ 350°	0.004835888
10°	30°~ 330°	0.037841512
20°	50°~ 310°	0.071640216
30°	70°~ 290°	0.099098832
40°	90°~ 270°	0.119916888
50°	110°~ 250°	0.134945232
60°	130°~ 230°	0.145327696
70°	150°~ 210°	0.152053952
80°	170°~ 190°	0.155822296
90°	190°~170°	0.078517492
	Total Area	1.000000000

Table 3 Quadrangles

2.2.-Plotting areas

Evaluating the formula with the values used by Don Davis [3] we obtain the following table (Table 4) where to obtain the values indicated in columns 3 and 4, the authors needed to include the values shown in Column No. 2[8] on the table shown below

$$0° = \frac{0.004835888}{2} = 0.00241794$$

$$10° = \frac{0.37841512}{8} = 0.004730$$

$$90° = \frac{0.078517492}{4} = 0.019630$$

Intervals In Ref. No. 2	Formula Limits	Area Except axial	Partials Sub-areas
0°	10°~ 350°	0.002418	0.002418
10°~ 350°	30°~ 330°	0.004730*4	0.01892
20°~ 340°	50°~ 310°	0.008955*4	0.03582
30°~ 350°	70°~ 290°	0.012387*4	0.049548
40°~ 320°	90°~ 270°	0.014990*4	0.05996
50°~ 310°	110°~ 250°	0.016868*4	0.067472
60°~ 300°	130°~ 230°	0.018166*4	0.072664
70°~ 290°	150°~ 210°	0.019007*4	0.076028
80°~ 280°	170°~ 190°	0.019478*4	0.077912
90°~ 270°		0.019630*4	0.07852
100°~ 260°	190°~ 170°	0.019478*4	0.077912
110°~ 250°	210°~ 150°	0.019007*4	0.076028
120°~ 240°	230°~ 130°	0.018166*4	0.072664
130°~ 230°	250°~ 110°	0.016868*4	0.067472
140°~ 220°	270°~ 90°	0.01499*4	0.05996
150°~ 210°	290°~ 70°	0.012387*4	0.049512
160°~ 200°	310°~ 50°	0.008955*4	0.03582
170°~ 190°	330°~ 30°	0.00473*4	0.01892
180°	350°~ 10°	0.002418	0.002418
			0.999968

Table 4 Partial Sub-areas

IV. USING THE MESUREMENTS WITH THE RINGS DIVISION TO GET THE Q

Once we get the ponderation factors showed like measurements sub-areas (Section 1.4), we can write two tables. One, with horizontal measurements and the other with the vertical. The column of the one octave (1/8), tabulated before will be only multiply by 4 instead of 8 because we will not use the diagonal measurements.

Table 5 represents the measurements taken expressed in power form multiplied by their corresponding surfaces. Known as L_w.

Angle	dB/Measured	P1/P2*	Area factor	P1/P2 *area
0° (Axial)	100	$1.00000*10^{10}$	0.001902651(1)	$1.902651*10^{7}$
10°~ 350°	99	$7.943282*10^{9}$	0.0001891804(4)	$6.010855*10^{7}$
20°~ 340°	98	$6.309577*10^{9}$	0.003726127(4)	$9.404110*10^{7}$
30°~ 330°	97	$5.011872*10^{9}$	0.005447234(4)	$1.092034*10^{8}$
40°~ 320°	96	$3.981072*10^{9}$	0.007002829(4)	$1.115151*10^{8}$
50°~ 310°	96	$3.981072*10^{9}$	0.008345647(4)	$1.328985*10^{8}$
60°~ 300°	95	$3.162278*10^{9}$	0.009434886(4)	$1.193428*10^{8}$
70°~ 290°	95	$3.162278*10^{9}$	0.010237451(4)	$1.294947*10^{8}$
80°~ 280°	94	$2.511886*10^{9}$	0.010728956(4)	$1.077997*10^{8}$
90°~ 270°	93	$1.995262*10^{9}$	0.010894468(4)	$8.694928*10^{7}$
100°~ 260°	90	$1.000000*10^{9}$	0.010728956(4)	$4.291583*10^{7}$
110°~ 250°	86	$3.981072*10^{8}$	0.010237451(4)	$1.630241*10^{7}$
120°~ 240°	79	$7.943282*10^{7}$	0.009434886(4)	$2.997759*10^{6}$
130°~ 230°	76	$3.981072*10^{7}$	0.008345647(4)	$1.328985*10^{6}$
140°~ 220°	75	$3.162278*10^{7}$	0.007002829(4)	$8.857956*10^{5}$
150°~ 210°	72	$1.584893*10^{7}$	0.005447234(4)	$3.453314*10^{5}$
160°~ 200°	69	$7.943282*10^{6}$	0.003726127(4)	$1.183907*10^{5}$
170°~ 190°	66	$3.981072*10^{6}$	0.001891804(4)	$3.012564*10^{4}$
180°	63	$1.995262*10^{6}$	0.001902651(1)	$3.796288*10^{3}$
	dBspl Hor.	by area factor	Total.	$1.035308*10^{9}$

Table 5 Horizontal measurements at 600 Hz.

* This value comes from

$$\frac{P1}{P2} = 10^{(\frac{dB}{10}}$$

For the vertical measurements we used a similar method as that in the horizontal but avoiding those values over the 0° and 180° axes. Once we get the summation of the horizontal and vertical average, L_w's, (column 5 of Tables 5 and 6) we introduce this data into the general formula of the axial Q, using the measurements above the axes

$$\overline{L_w} = 10 * log\left(10^{\frac{L_{p1}}{10}} + 10^{\frac{L_{p2}}{10}} ... + 10^{\frac{L_{pn}}{10}}\right) - 10 * \log(n)$$

Where n is the number of measurements.

Angle	dB/Measured	P1/P2	Area Factor	P1/P2 *area
10°~ 350°	100	$1.00000*10^{10}$	0.001891804(4)	$7.567218*10^{7}$
20°~ 340°	100	$1.00000*10^{10}$	0.003726127(4)	$1.490451*10^{8}$
30°~ 330°	99	$7.943282*10^{9}$	0.005447234(4)	$1.730757*10^{8}$
40°~ 320°	98	$6.309573*10^{9}$	0.007002829(4)	$1.767395*10^{8}$
50°~310°	96	$3.981072*10^{9}$	0.008345647(4)	$1.328985*10^{8}$
60°~ 300°	96	$3.981072*10^{9}$	0.009434886(4)	$1.502438*10^{8}$
70°~ 290°	94	$2.511886*10^{9}$	0.010237451(4)	$1.028613*10^{8}$
80°~ 280°	92	$1.584893*10^{9}$	0.010728956(4)	$6.801700*10^{7}$
90°~ 270°	90	$1.000000*10^{9}$	0.010894468(4)	$4.357787*10^{7}$
100°~260°	87	$5.011872*10^{8}$	0.010728956(4)	$2.150886*10^{7}$
110°~ 250°	85	$3.162278*10^{8}$	0.010237451(4)	$1.294947*10^{7}$
120°~ 240°	82	$1.584893*10^{8}$	0.009434886(4)	$5.981315*10^{6}$
130°~ 230°	78	$6.309573*10^{7}$	0.008345647(4)	$2.106299*10^{6}$
140°~ 220°	74	$2.511886*10^{7}$	0.007002829(4)	$7.036124*10^{5}$
150°~ 210°	68	$6.309573*10^{6}$	0.005447234(4)	$1.374789*10^{5}$
160°~ 200°	66	$3.981072*10^{6}$	0.003726127(4)	$5.933592*10^{4}$
170°~ 190°	64	$2.511886*10^{6}$	0.001891804(4)	$1.900799*10^{4}$
	dBspl Vert.	By area factor	Total.	$1.115596*10^{9}$

Table 6 Vertical iteration 600Hz

Using the formula to calculate the Axial Q with the values

$$\overline{L_w} = 10 * \log(1.035308 * 10^9 + 1.115596 * 10^9) - 10 * \log(1)$$

L_w(Average)= 93.33 $dBspl$

$$Q = 10^{\left(\frac{100-93.33}{10}\right)} = 4.65\ (600Hz)$$

Comparing the value gotten with the chart of the factory, (See Figure 7), taking in count that we measured at 1 meter and JBL did at 4 meters, we can see there is a good approximation keeping a proportional ratio with the areas of each case.

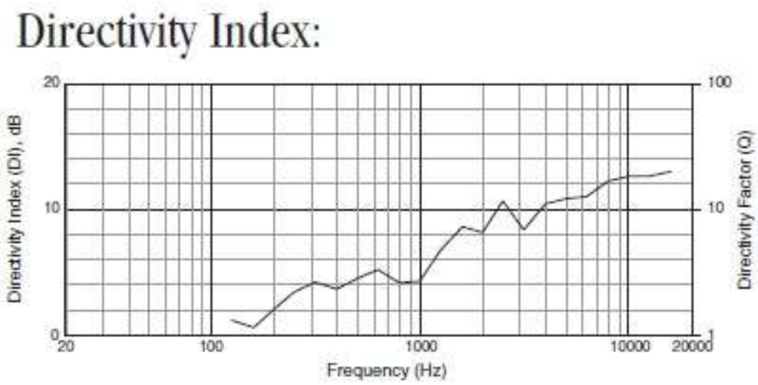

Figure 7 Directivity Index of the JBL Vertec 4886 System

V. CONCLUSION

As the paper indicates, both methods of dividing the sphere in rings or lunar poles are acceptable to study the acoustics of sound sources. In this paper, the authors wanted to highlight the differences between these methods to alleviate to the newcomers of the study of acoustics some of the confusions that we have observed in new practitioners. Hence, the expansion on the mathematical details in the use of the formulas.

We concluded also that the similarity of the results between the manufacturer chart of specification and our experiment shows that the method developed by Don Davis to determine the Q factor is an accurate approximation. In addition, this work demonstrate that the adjustment of the distances carried out on this experiment still maintains the constant proportion of sound radiation over increasing areas. The contribution of G. L. Wilson [4] was invaluable in the evaluation of the work of Don Davis [3].

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