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Comparison between Arithematic and Geometric Progression of Rounding Off Errors in Population Projection of Katsina State

TAJUDEEN ALANI WILLIAMS[1], USMAN KABIR KURFI[2]
[1,]*Department of Mathematics, Federal College of Education, Katsina, Nigeria.*
[2,]*Department of Mathematics and Statistics, Hassan Usman Katsina Polytechnic, Katsina, Nigeria.*

Abstract: *Population projection is a mathematical equation that calculates the estimated growth rate or change of future populations based on current populations. The study was to determine the rate of population growth in Katsina State, round off the computed rate at various decimal places and ascertain whether rounding off errors affect the population projection figures and also compare between the two models. The study makes use of arithmetic and geometric mean methods to see the effect of round off error on population projection. This involves the computation of average annual growth rate 'r' and the results were approximated at various decimal places. The approximated values were substituted into the arithmetic and geometric models to compute the projection and plot the graphs to determine the effect of rounding of errors. The computed population growth rate was 0.03638 and 0.02946 for arithmetic and geometric models respectively which shows 36% and 29% increase in population from 2006 to 2019. This growth rate was used to determine the population projections for the time period selected in the study and it was observed that the round off error affect the population figures and the 2 decimal place round off growth rate gave the best estimate. Hence, it was concluded that the 2 decimal place round off provided the projections with the least error. Therefore, it was recommended that a 2-decimal place round-off error should be used in population projections for arithmetic model.*

Keywords: *Rounding off error, Population, Arithmetic and geometric mean methods, Estimate*

I. Introduction

Globally, Nigeria is among the ten top countries with the highest population, in fact, the seventh among the countries with the highest population in the world (United Nations, 2009 & Internetworldstats, 2021). The population of Nigeria is increasing rapidly and this rapid increase may be due to the effect of three important factors - birth rate, death rate and higher net migration. Migration however, has played a negligible role in Nigerian population increase. Therefore, it will not be given prominence in this write up. If an increasing population is brought about as a result of an increase in birth rate, there will be higher percentage of young people and children in the population and if it is by a decrease in death rate, then, there will be an increase in the total number of old people in the population. Furthermore, if an increase in population is brought about as a result of a higher net migration, then, there will be a larger number of people between ages of 16 and 50 years. Nigeria population is increasing mostly due to the effect of the first factor, that is, high birth rate. For instance, Crude Birth Rate (CBR) was 39.0 and 44.6 in 1990 and 1991 respectively (NPC, 2000) while it was 42.0 in 2003 (NDHS, 2003). All these buttress the fact that there is high CBR in Nigeria which shows that the population has gathered momentum and it will continue to increase for some time even if there is a change

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favorable towards family planning and birth control. Increasing population at the expense of socio-economic development is inimical to people's well-being and development. An increasing population has consequences and implications, most especially for a country like Nigeria and this is the thrust of this study.

Nigeria has a growing population and what can also be referred to as an increasing population. The census figure of 1991 put Nigerian population at about 89 million people with the growth rate of 2.82. The Nigeria Demographic and Health Survey (NDHS, 2003 & 2008) put the total fertility rate at 5.7 percent as against that of 1999 NDHS which was 5.2 percent. Going by 2006 Nigerian National Population Census, Nigeria had a population of one hundred and forty million, three thousand five hundred and forty two (140,003,542) with a growth rate of 3.02 percent per annum (National Bureau of Statistics, 2009). The population is capable of doubling itself in less than twenty three years. In addition, the United Nations estimate of 2009 put the Nigerian total population at one hundred and fifty one million, thirty thousand and four hundred (151,030,400). Therefore, Nigeria is the most populous country in Africa and also the most populous among the black nations of the world.

Population in Katsina State

Katsina State was created out of the defunct Kaduna State on 23rd September, 1987 and the former Katsina Province of old Northern Nigeria. The State comprises of Katsina and Daura Emirates, it borders Kaduna State to the south, Jigawa and Kano States to the east, Zamfara State to the west and shares an international border with Republic of Niger to the North. It occupies an area of about 24,192 square kilometers with an estimated population of about 5.8 million people as per 2006 projection.

Population Projection

Population projection is a mathematical equation that calculates the estimated growth rate or change of future populations based on current populations. Government use population projections for planning in public health, housing, school and hospital constructions and such information also aids business and marketing. Population projection is an estimate of the number of people expected to be alive at a future date, based on assumptions of population size, births, deaths, and migration. Population projections are useful tools for program planning and policy dialogue. It is a "best-guess" calculation of the expected number of people that will be alive at a future date based on the current population size, expected births, deaths, and migration Yadav (No date).

Population projections are used for the calculations of future birth rate, death rate and migration of population based on their past and present conditions. They are in general purely formal calculations, developing the implications of the assumptions that are made. Projections made for the whole country are called total projections. But when projections are made for a region, state, district or ethnic group, they are called regional or sectorial projections.

Demographers are frequently called to produce population information when census and related data are required. An estimated population gives information about present or past populations not based on a census or population register. Information about the future population is referred to as either a projection or a forecast and can be differentiated according to the expected likelihood of their outcomes.

Reviews

According to 2006 Population Census, the Nigerian growth rate was 3.02% and this shows that it will double in less than 22 years. According to Jones & Lopez (2006), birth rate is the number of individuals born in a given year per 1000 in the population. If 40 births occur in a given year per 1000 individuals in the population, the birth rate is 40 and this is 4%. It should be noted that the younger the number of individuals in a population, the higher the birth rate and consequently the faster the population growth. There are many factors that have generated increase in birth rate.

1. One of them is improved medical services and facilities. Compared to many decades ago, there are better though not adequate medical services and facilities. Better drugs and immunization services which have reduced greatly infant, child, maternal and other forms of mortality and consequently high birth rate. Early marriage most especially in the northern part of Nigeria. Early marriage tend to lead to high birth rate because women will have opportunity of having many children due to long child bearing/reproductive years while postponement of marriage because of educational aspirations will reduce birth rate.
2. The increase in material wellbeing of some families. When people are materially well off, they give little thought to the number of children to have. Wealth has encouraged many people to have large families and thus result to high birth rate. Moreover, old age security also encourages people to have large families. People want to have many children base on the fact that these children would provide for them when they are old and cannot engage in productive activities again. They believe the more the children the more the guarantee of better life at old ages. They strife and try to have more and more children.
3. In Nigeria, religion, superstition and customs have encouraged population growth. All these favor large families and discourage the practice of family planning. Many religion and customs also allow polygamy. For instance, Islam favors polygamy and allows men to have up to four wives each.
4. Death rate is also worth mentioning as a factor in population growth in Nigeria. Many of the factors mentioned above can also be held responsible for low death rate that is generating increase in population growth. Factors that affect death rate include improvement in medicine, dietary standard, hygiene, increase in the levels of standard of living and literacy etc.

According to Macunovich (2000), population projections can be used for planning food and water usage, and public services such as health and education. Zoning and other demographic boundaries rely on population projections as well. Businesses use population projections for store location planning and marketing and projections can also affect federal and state funding.

According to United Nation (2015), the world population was projected to increase by more than one billion people within the next 15 years, approaches nearly 10 billion in 2030, and will increase further to 9.7 billion in 2050 and 11.2 billion by 2100 while climate change and socioeconomic factors will continue to pose a challenge to demographers. A need for more accurate population projection models becomes more crucial and more valuable for everyone.

Tom (2011) said that based on such projections, we can raise our understanding of the determinants of population change such as what impact would a 20% decline in birth rates have on a country's population size and age structure in the next 50 years? They can also be used to provide information on possible future scenarios since we cannot "see" into the future. Population projections are essential for planning at the national, regional and state levels in both the private and public sectors. For efficient allocation of the scarce resources, planners and policy makers should know the future size and structure of the country's population as well as its characteristics.

Planning in any sector of the economy therefore requires information about the future size and structure of the population of the area. This is because the changes in population size and composition have many social, economic, environmental and political implications. Population projections often serve as a basis for producing other projections (e.g., households, school enrolments, income, labour force, etc). The most common sub regional projection methods are non-component; Trend Extrapolation, Auto-Regressive Integrated Moving Average (ARIMA), Comparative, Regression, Economic Base, Housing Unit, Land-use allocation, Averaged Projections and Shortcut Cohort methods. The Component projection methods are; Simple Component, Cohort-Component, Micro simulation and Integrated Projection models.

Objective

1. To determine the rate of population growth in Katsina state.
2. To round off the computed rate to various decimal places using both Arithmetic and Geometric models.
3. To ascertain whether rounding off errors affect the population projection figures using arithmetic and Geometric models.
4. To find out which among the arithmetic model for various approximated rates is best for projection population.
5. To compare the best between Arithmetic and Geometric models used in population projection.

Research Hypotheses

The following hypotheses were used to guide the study:

H_{01}: There is no increase in the annual rate of population growth in Katsina State.
H_{02}: No rounding off error affects population projection using arithmetic and Geometric model
H_{03}: There are no differences among the various approximated rates.
H_{04}: None of the models is termed the best in population projection.

II. Material and Methods

This paper is indeed analytic in nature and secondary method of data collection was applied in sourcing the data from the National Population Commission (NPC) based on the Katsina State estimated population as of the year 1999 to 2006**.** The target population of this paper is the entire estimated population of the Katsina State from 1991 to 2006. The paper makes use of arithmetic mean method to see the effect of rounding off error in population projection.

Population projection:

Population projection is an exercise of calculating the future values of a population. A population projection is an extrapolation of historical data into the future. According to a UN Study in Yadav (No date), population projections are calculations which show the future course of fertility, mortality and migration. Population projections are calculations of future birth rate, death rate and migration of population based on their past and present conditions.

A population projection is:

- An extrapolation of historical data into the future
- An attempt to describe what is likely to happen under certain explicit assumptions about the future as related to the immediate past
- A set of calculations, which show the future course of fertility, mortality and migration depending on the assumptions used

Uses of Population Projections

National Records of Scotland (No Date) said it is becoming very important to have high quality statistics of the population and the projections of the population for long term and short-term national policy development and planning in other to provide public services in different geographic areas. They are used for:

- Sectorial plans and allocation of funds
- provision of nurseries or day care centers
- local and national policy
- housing and land use planning
- health care planning
- modeling and projecting health care indicators

- weighting surveys
- benchmarking other projections and as a control for smaller area projections
- teacher workforce models both at a national and local level
- looking at the implications of an ageing population and
- making national and international comparisons, etc.

Techniques

Population projection techniques refer to the various procedures for arriving at the future size and the age-sex composition of the population at specific points in time. The most common sub regional projection methods are non-component; Trend Extrapolation, Auto-Regressive Integrated Moving Average (ARIMA), Comparative, Regression, Economic Base, Housing Unit, Land-use allocation, Averaged Projections and Shortcut Cohort methods. The Component projection methods are; Simple Component, Cohort-Component, Micro simulation and Integrated Projection models. The various methods of population projections could be broadly classified into;

(a) Mathematical projections method

(b) cohort- component method (Ayhan, 2018)

The distinction between the two methods of population projections lies on the nature of assumptions made about the pattern of change in the different elements of population dynamics.

Mathematical Methods: The use of the mathematical methods involves application of a mathematical equation to derive projections of the total population on the basis of total population data available from one or more censuses. The mathematical methods are: Arithmetic growth and Geometric growth but we only take the arithmetic growth model to see the effect of rounding of off errors. The method also involves the computation of average annual growth rate of 'r' and the approximation of the results at various decimal places. The approximated values will be substituted in arithmetic growth model to compute population projection and plot the projection graph to determine the effect of rounding off errors

Arithmetic Growth Model: This method assumes that population growth is linear or follows an arithmetic progression. i.e, that there is a constant amount of increase per unit of time.

Projection equation: $P_t = P_o(1 + rt)$

where

P_T = size of the population in year T

P_o = size of the population in the base year

r = average annual rate of growth$= \frac{(P_T - P_o)}{T_o P_o}$

T_o = length of the time interval, (highest – lowest)

Geometric Growth Model

Under this method, it is assumed that population growth follows a geometric progression i.e. The population in succeeding years increase or decreases at a constant proportion or percentage of the population in the previous year.

Projection equation: $p_t = p_0(1 + r)^t$

$r = \left(\sqrt[T]{\frac{P_T}{P_0}}\right) - 1$

III. Results and Discussion

Table 1: Population figures of Katsina State (1991 and 2006 Census)

YEAR	1991	2006
No. of Population	3,753,133	5,801,584

Table 1 above depicts the population figures of Katsina state from which it shows that the total figure during 1991 census was 3,753,133. However, the population figures after 2006 census exhibit about 35.3% increase with reference to 1991 census. Considering the period of 15 years, we can say that there is a rapid population growth in the state.

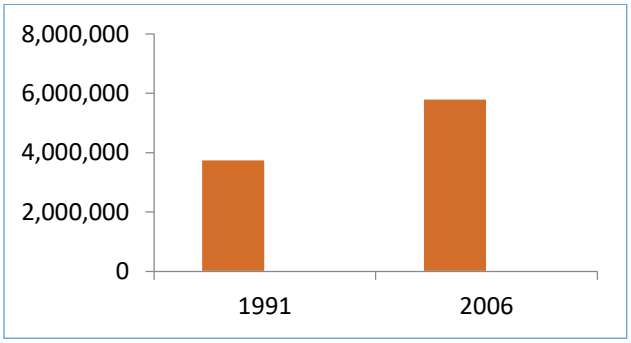

Fig 1: Distribution of population figures in 1991 and 2006

Rate Growth Computation (r) for AP and GP

AP $r = 0.03638$

GP $r = 0.02946$

Table 2: Population growth rate in Katsina State

	Round off to 1dp	Round off to 2dp	Round off to 3dp	Round off to 4dp
AP	0.0	0.04	0.036	0.0364
GP	0.0	0.03	0.029	0.0295

Table 2 above highlighted the population growth rate at various round off figures. Considering the mathematics behind rounding off of figures and decimal truncation, we have an evidence to believe that the two-decimal place round off is the most appropriate for the value r. We shall therefore say the 0.04 for arithmetic and 0.03 for geometric model is the rounded off figure with the least error of approximation.

A.P Projection equation: $P_t = P_o(1 + rt)$ and G.P Projection equation: $p_t = p_0(1 + r)^t$

Where t is the different between year Yi and 2006 i.e. t for year $1 = 2007 - 2006 = 1$

$$\therefore t = 1 \dots \dots \dots \dots \dots \dots 13$$

Table 3: Population projection figures and rates for Arithmetic model

YEAR	P_0	t	r 1dp	r 1dp P_t	r 2dp	r 2dp P_t	r3d	r3d P_t	r 4dp	r 4dp P_t
2007	5801584	1	0	5801584	0.04	6033647	0.036	6010441	0.0364	3889747
2008	5801584	2	0	5801584	0.04	6265711	0.036	6219298	0.0364	4026361
2009	5801584	3	0	5801584	0.04	6497774	0.036	6428155	0.0364	4162975
2010	5801584	4	0	5801584	0.04	6729837	0.036	6637012	0.0364	4299589
2011	5801584	5	0	5801584	0.04	6961901	0.036	6845869	0.0364	4436203
2012	5801584	6	0	5801584	0.04	7193964	0.036	7054726	0.0364	4572817
2013	5801584	7	0	5801584	0.04	7426028	0.036	7263583	0.0364	4709431
2014	5801584	8	0	5801584	0.04	7658091	0.036	7472440	0.0364	4846045
2015	5801584	9	0	5801584	0.04	7890154	0.036	7681297	0.0364	4982659
2016	5801584	10	0	5801584	0.04	8122218	0.036	7890154	0.0364	5119273
2017	5801584	11	0	5801584	0.04	8354281	0.036	8099011	0.0364	5255887
2018	5801584	12	0	5801584	0.04	8586344	0.036	8307868	0.0364	5392501
2019	5801584	13	0	5801584	0.04	8818408	0.036	8516725	0.0364	5529116

The population growth rate was found to be 0.03638 and table 3 above shows the projection figures computed from the population growth rate at different round off values. The projection figures computed with 1 decimal place round off of the growth rate is found to be static, because the projected figure for all the years remained unchanged. The computed projections at 4 decimal place rate is misleading as all the projected values are less than the population of the base period. However, the 2 decimal and 3 decimal place computed projections are reliable projections with the least computational error.

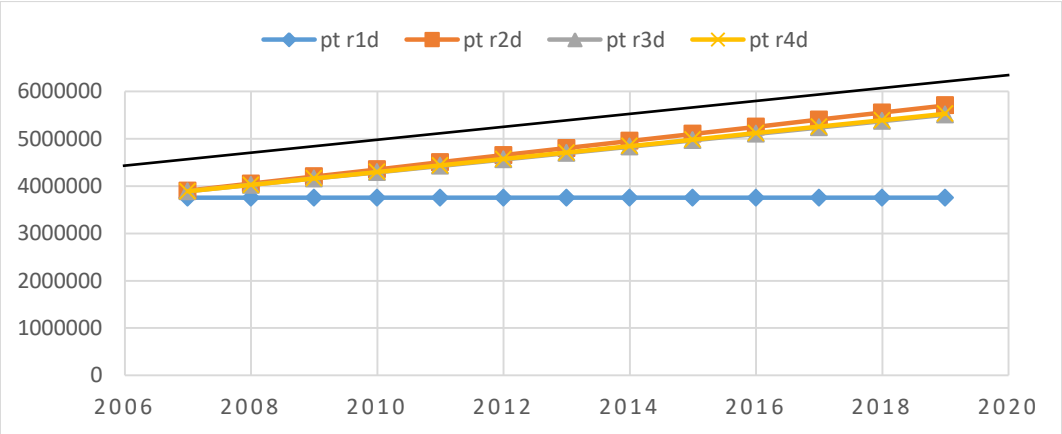

Fig 2: Series plot for population projections using Arithmetic model

The figure above shows the projections at various round-off figures of the growth rate. The thick line represents the observed figures while the other lines are the projections. We observed that the projections obtained using 2 decimal figure of the growth rate gave the best estimates of the observed figures, and hence we can say that this projection has least error content.

				r 1dp		r 2dp		r3d		r 4dp
										P_t
YEAR	P_0	t	r 1dp	P_t	r 2dp	P_t	r3d	P_t	r 4dp	
2007	5801584	1	0	5801584	0.03	5975632	0.029	5969830	0.0295	5972731
2008	5801584	2	0	5801584	0.03	6154900	0.029	6142955	0.0295	6148926
2009	5801584	3	0	5801584	0.03	6339547	0.029	6321101	0.0295	6330320
2010	5801584	4	0	5801584	0.03	6529734	0.029	6504413	0.0295	6517064
2011	5801584	5	0	5801584	0.03	6725626	0.029	6693041	0.0295	6709317
2012	5801584	6	0	5801584	0.03	6927395	0.029	6887139	0.0295	6907242
2013	5801584	7	0	5801584	0.03	7135217	0.029	7086866	0.0295	7111006
2014	5801584	8	0	5801584	0.03	7349273	0.029	7292385	0.0295	7320781
2015	5801584	9	0	5801584	0.03	7569751	0.029	7503864	0.0295	7536744
2016	5801584	10	0	5801584	0.03	7796844	0.029	7721476	0.0295	7759078
2017	5801584	11	0	5801584	0.03	8030749	0.029	7945399	0.0295	7987970
2018	5801584	12	0	5801584	0.03	8271672	0.029	8175815	0.0295	8223615
2019	5801584	13	0	5801584	0.03	8519822	0.029	8412914	0.0295	8466212

Table 4: Population projection figures and rates for Geometric model

The population growth rate was found to be 0.02946 and table 4 above shows the projection figures computed from the population growth rate at different round off values. The projection figures computed with 1 decimal place round off of the growth rate is found to be static, because the projected figure for all the years remained unchanged. The computed projections at 4 decimal place rate is misleading as all the projected values are less than the population of the base period. However, the 2 decimal and 3 decimal place computed projections are reliable projections with the least computational error.

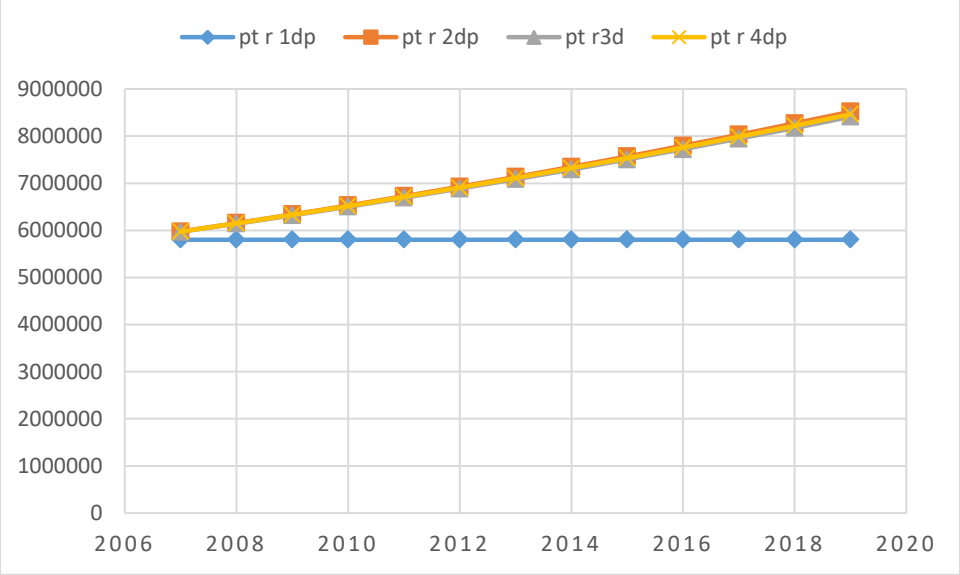

Fig 3: Series plot for population projections using Geometric model

The figure above shows the projections at various round-off figures of the growth rate. The thick line represents the observed figures while the other lines are the projections. We observed that the projections obtained using 2 decimal figure of the growth rate gave the best estimates of the observed figures, and hence we can say that this projection has least error content.

IV. Conclusion

The computed population growth rate for arithmetic model and geometric model was found to be 0.03638 and 0.02946 respectively, which means that there is a 36% increase in population from 2006 to 2019 for arithmetic model and 29% increase in population from 2006 to 2019 for geometric model. The growth rate was used to determine the population projections for the time period selected for the study, and it was observed that the 2 decimal place round off rate of growth gave the best estimates of the observed population figures for the both models since it provided the projections with the least error. Therefore, there is enough evidence to say that the round off error affect the population figures and the arithmetic model has higher difference between decimal place compare to geometric model.

V. Recommendations

It is highly recommended to use a 2-decimal place round-off of the population growth rate while computing population projections using arithmetic model and geometric model, as it truncates the error of estimation to the least and it provides estimate with more precise values.

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